UNIVERSITY OF CALIFORNIA

College of Engineering
Department of Electrical Engineering and
Computer Sciences

Professor Tse Spring 2004

EECS 126 — Midterm #2

- **Closed book, no notes, no calculators.**
- **Please write ALL of your solutions in the blue book. The question sheet itself will be collected at the end of the exam.**
- **Please show your steps clearly. True/false answer without explanation gets no mark.**
- **BE SURE TO WRITE YOUR NAME ON YOUR BLUE BOOK!!!!**

Formula for pdf of $N(0,1)$ rv: $f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$

Question	**Points possible**	**Your points**
1	16	
2	20	
3	16	
4 – Part I	17	
4 – Part II	31	
Total	100	

Problem 1 (16 points)

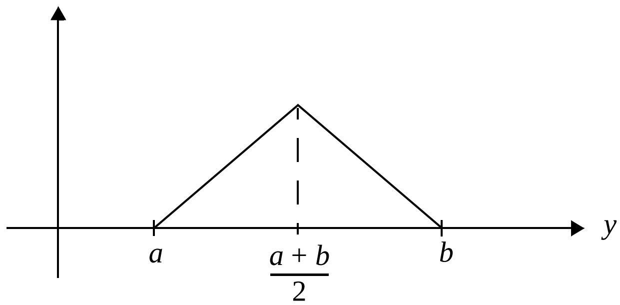
[7 pts.] a. X is an exponential rv with parameter μ:

$$f_X(x) = ae^{-\mu x} \qquad x \geq 0.$$

[3 pts.] (i) Compute a.

[4 pts.] (ii) Sketch the pdf for 2 different values of $\mu_1, \mu_2, \mu_1 > \mu_2$ on the same plot.

[9 pts.] b. Y has a pdf as follows:



[5 pts.] (i) Sketch the pdf of $Z = cY + d$, where $d > 0$, on the same as plot as pdf of Y. Consider all possible cases.

[4 pts.] (ii) Find the mean and variance of Z.

Problem 2 (20 points)

Let X and Y be two independent exponential random variables with parameters μ_X and μ_Y, respectively.

[5] a) Find the pdfs of $\max(X, Y)$ and $\min(X, Y)$.

[4] b) Find the pdf of $X + Y$.

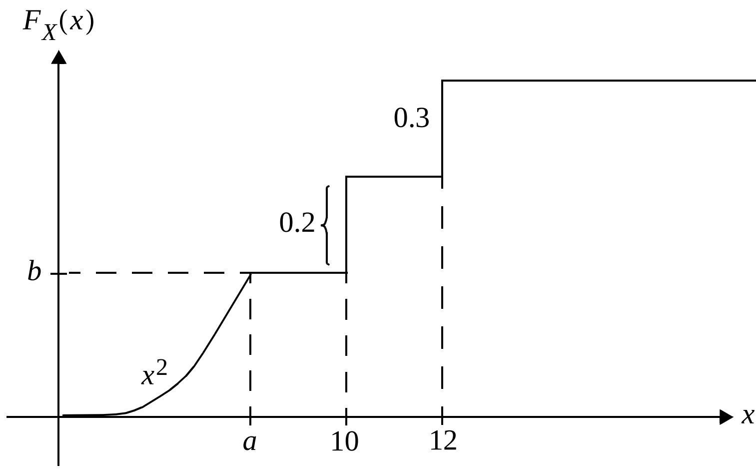
[4] c) Find the pdf of X conditional on $X + Y = a$.

[4] d) Find the pdf of X conditional on $X \geq x$.

[3] e) Using your answer to part (d), explain why the exponential distribution is sometimes said to be "memoryless."

Problem 3 (16 points)

The cumulative distribution function of a rv X looks as follows:



(Note: Not to scale)

[2] a) Is this a continuous or a discrete rv? Or neither? Explain.

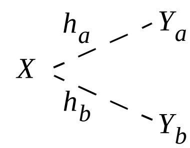
[4] b) Can you determine the values of a and b from the given information? If so, find their values.

[5] c) Suppose you want to generate this rv on MATLAB but you only have access to rand($\cdot$), which generates a uniform rv in [0,1]. Explain how you would simulate X.

[5] d) Suppose now you don't even have access to rand($\cdot$) but you do have access to the routine Bern(p), which generates a Bernoulli random variable with parameter p. Explain how you would use this routine to generate X.

Problem 4 (Part I – 17 points; Part II – 31 points)

Consider a wireless communication system with a single transmit antenna and two receive antennas. The channel is described by



$$Y_a = h_a X + W_a$$
$$Y_b = h_b X + W_b$$

where X is the transmitted symbol; Y_a, Y_b are the received symbols in antenna a and b, respectively; and W_a, W_b are independent $N(0, \sigma^2)$ noise at the two antennas independent of X. h_a and h_b are the channel gains.

Part I – In this part you can assume that h_a and h_b are determinative and known numbers.

[6 pts.] **a.** Suppose you transmit $X = \pm a$, equiprobable. Find the MAP detection rule of X based on Y_a alone and the error probability in terms of the signal-to-noise ratio.

[8 pts.] **b.** Repeat part (a), but now base the detection of X based **on both** Y_a and Y_b.

[3 pts.] **c.** In a wireless environment, sometimes h_a or h_b can be very small due to channel fading. Based on your answers to part (a) and (b), what do you think is the advantage of having two receive antennas?

Part II – So far we have assumed that h_a, h_b are known numbers, but suppose now that they too are random variables:

$$h_a, h_b \sim N(0, 1),$$

independent and independent of everything else. This models the random fading and the fact that the receiver does not know what the values of h_a and h_b are.

[6 pts.] **a.** Compute the joint distribution of Y_a and X in the communication scheme of Part I(a). Do you think it is still a good communication scheme when the channel gains are random and unknown?

[6 pts.] **b.** What about the communication scheme in PartI(b)?

[11 pts.] **c.** Consider now another scheme. For now let us focus on the scenario when there is a single receive antenna, a. The communication scheme involves sending information over two symbol times:

$$Y_{ia} = h_a X_i + W_{ia}, \quad i = 1, 2$$

where X_i is the transmitted symbol at time i and Y_{ia} is the received symbol at time i at antenna a. $W_{ia} \sim N(0, \sigma^2)$. The noise is independent of each other and of the transmitted symbol.

The scheme transmits one bit of information and works as follows:

To transmit a '0', we send

$$X_1 = 1, X_2 = 0.$$

To transmit a '1', we send

$$X_1 = 0, X_2 = 1.$$

Both hypotheses are equally probable.

[3 pts.] (i) Do you think this scheme can convey any information? Give some intuition.

[8 pts.] (ii) Find the MAP rule for deciding whether a '0' or a '1' is transmitted.

[8 pts.] **d.** Extend the MAP rule in Part II(c)(ii) to the case when there are two receive antennas, antenna 'a' and 'b'. Intuitively, do you think having two receive antennas still provide some benefit in this setting where the channel gains are unknown?