

Department of EECS - University of California at Berkeley
EECS126 - Probability and Random Processes - Fall 2003
Midterm No. 1: 10/3/2003

Name and SID:

There are six questions. Answer on these sheets. Show your work. Good luck.

Question 1 (15%). Is it true that

$$P(A \cap B \cap C) = P[A | B]P[B | C]P(C)?$$

If true, provide a proof; if false, provide a counterexample.

Question 2 (15%). Describe the probability space $\{\Omega, \mathcal{F}, P\}$ that corresponds to the random experiment “picking five cards without replacement from a perfectly shuffled 52-card deck.”

Question 3 (20%). Choose X in $[0, 1]$ as follows. With probability 0.2, $X = 0.3$; with probability 0.3, $X = 0.7$; otherwise, X is uniformly distributed in $[0, 0.5] \cup [0.6, 0.9]$. (a). Plot the c.d.f. of X ; (b) Find $E(X)$; (c) Find $var(X)$; (d) Calculate $P[X \leq 0.8 \mid X \geq 0.4]$.

Question 4 (15%). Let (X, Y) be the coordinates of a point picked randomly and uniformly in $[0, 1]^2$. Calculate $P[X + 2Y \leq 1 \mid 2X + Y \leq 1]$.

Question 5 (15%). Let X be a random variable that is exponentially distributed with mean 1. Calculate $P[X \in [1, 4] \mid X \in [3, 5]]$.

Question 6 (20%). Let (X, Y) be the coordinates of a point picked uniformly in $\{(x, y) \in \mathfrak{R}^2 \mid x^2 + y^2 \leq 1\}$. Calculate $E(|X|)$.

(*Hint:* First find f_Y where $Y = |X|$. To do that, look at the set of outcomes such that $Y \in (x, x + dx)$ and determine its probability.)