EECS123, Spring/1994
Final Exam
Professor Vetterli

Problem #1

Problem 1: Linear Phase Filters

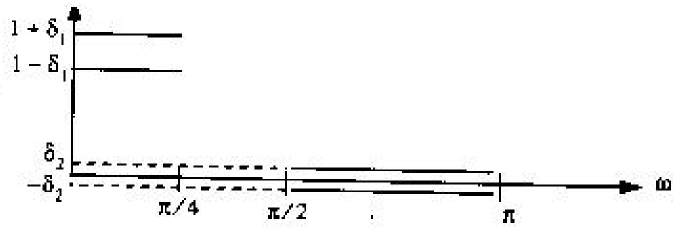
a. Consider Ha(z) = 1 + 2z^(-1) + 3z^(-2).
Show that Ga(z) = Ha(z)Ha(z^-1) is a linear phase filter. What type is it, and what is the phase? Compute the coefficien

b. Consider an arbitrary FIR filter Hb(z) with real coefficients and show that Gb(z) = Hb(z)Hb(z^-1) is a linear phase f

c. Show that Ga(e^jw) as well as Gb(e^jw) are always non-negative for any w.

d. Consider Hc(z) = [1-e^(j*theta)z^(-1)][1-e^(-j*theta)z^(-1)] and the associated Gc(z) = Hc(z)Hc(z^-1).
Sketch Gc(e^jw) for theta = 3pi/4 and w = [0, 2pi].

e. Assume the following desired specifications for a lowpass filter:



1 + δ1
1 − δ1
δ2
−δ2
π/4
π/2
π
ω

Assume two linear phase filters are designed to meet the specifications, one using windowing and the other optimal Pa

Sketch roughly their respective magnitude responses with respect to the desired specifications. What can you say abou

Problem #2

Consider the sequence x(n) = 1/(2^n), n>=0, 0 else.

a. Give the z-transform X(z), a pole-zero plot(with the ROC) and the discrete-time Fourier transform (DTFT) X(e^jw).

b. We derive the coefficients

$\tilde{X}[k]$

of a discrete-time Fourier series (DTFS) by sampling X(e^jw) at w = 2pi*k/N,

$$\tilde{X}[k] = X(e^{j2\pi k/N}) \quad k = 0 \ldots N-1$$

Give the DTFS coefficients for N=4.

c. Give an expression for the time-domain periodic signal

$\tilde{x}[n]$

which has DTFS coefficients

$$\tilde{X}[k] = X(e^{j2\pi k/N}) \quad k = 0 \ldots N-1$$

from (b) above. (Hint: It is easiest to do this in time-domain without using inverse DTFS.) Is there time-domain aliasin
period.

d. Consider now a sequence y[n] which is the impulse response of a general stable and casual IIR filter. A periodic sig

$\bar{y}[n]$

is obtained as

$$\tilde{y}[n] = \sum_{l=-\infty}^{\infty} y(n+lN)$$

While there is time-domain aliasing, show that

$\bar{y}[n]$

remains bounded , or

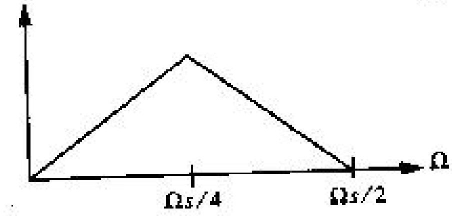
$|\tilde{y}[n]| < A < \infty$

where A is some positive constant. (Hint: This is easiest by going to frequency domain. Use the fact that stable and cau

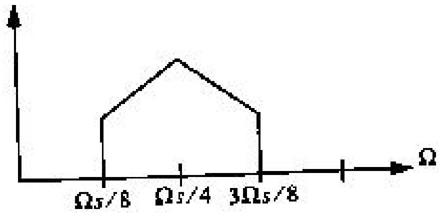
Problem #3

Assume a real signal with the spectrum given in (i). The spectrum given in (ii) is desired.

(i)



(ii)



Assume first sampling at

Ω_s

with no input filtering:

a. Give the magnitude specification of the discrete-time filter Hd(e^jw), assuming it is easier to get sharp cut-offs in th
continuous-time reconstruction filter.

b. Give the magnitude specifications of the continuous-time reconstruction filter

$H_c(\Omega)$

including transition bands.

c. In order to design the filter in (a), above, design a lowpass filter first, and modulate it adequately. What are the speci
modulation function?

Now, the sampling frequency can be changed.

d. Still assuming no filter is used before sampling, what is the minimum possible sampling frequency so that the desire
of

$H_d(e^{j\Omega})$

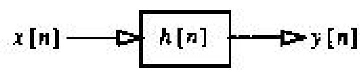
and

$H_r(\Omega)$

change?

Problem #4

Consider an LTI system with causal, finite impulse response h[n] with length P:



And suppose x[n] has length L < infinity.

a. Write down the N*N convolution matrix, Ch, so that

$\hat{y} = C_k \hat{x}$,

where y[n] = x[n] * h[n] (linear convolution)

$\hat{y} = (y[0], \ldots, y[N-1])^T$, and $\hat{x} = (x[0], \ldots, x[L-1], 0, \ldots, 0)^T$

Use the smallest possible value for N. Give N as a function of L and P.

b. We wish to recover x[n] from y[n]:

Again, we can write

$\hat{x} = C_g \hat{y}$.

What is the relationship between Cg and Ch? Find Cg in terms of D and /\h, where D is the N-point DFT matrix and

$C_h = D^{-1} \Lambda_h D$

c. Call H[k] and G[k] the elements on the diagonal of

Λ_h and Λ_g

respectively.

Assume H[k] != 0, k = 0,..., N-1. Find G[k] in terms of H[k] for k = 0...N-1. What is g[n] in terms of H[k]?

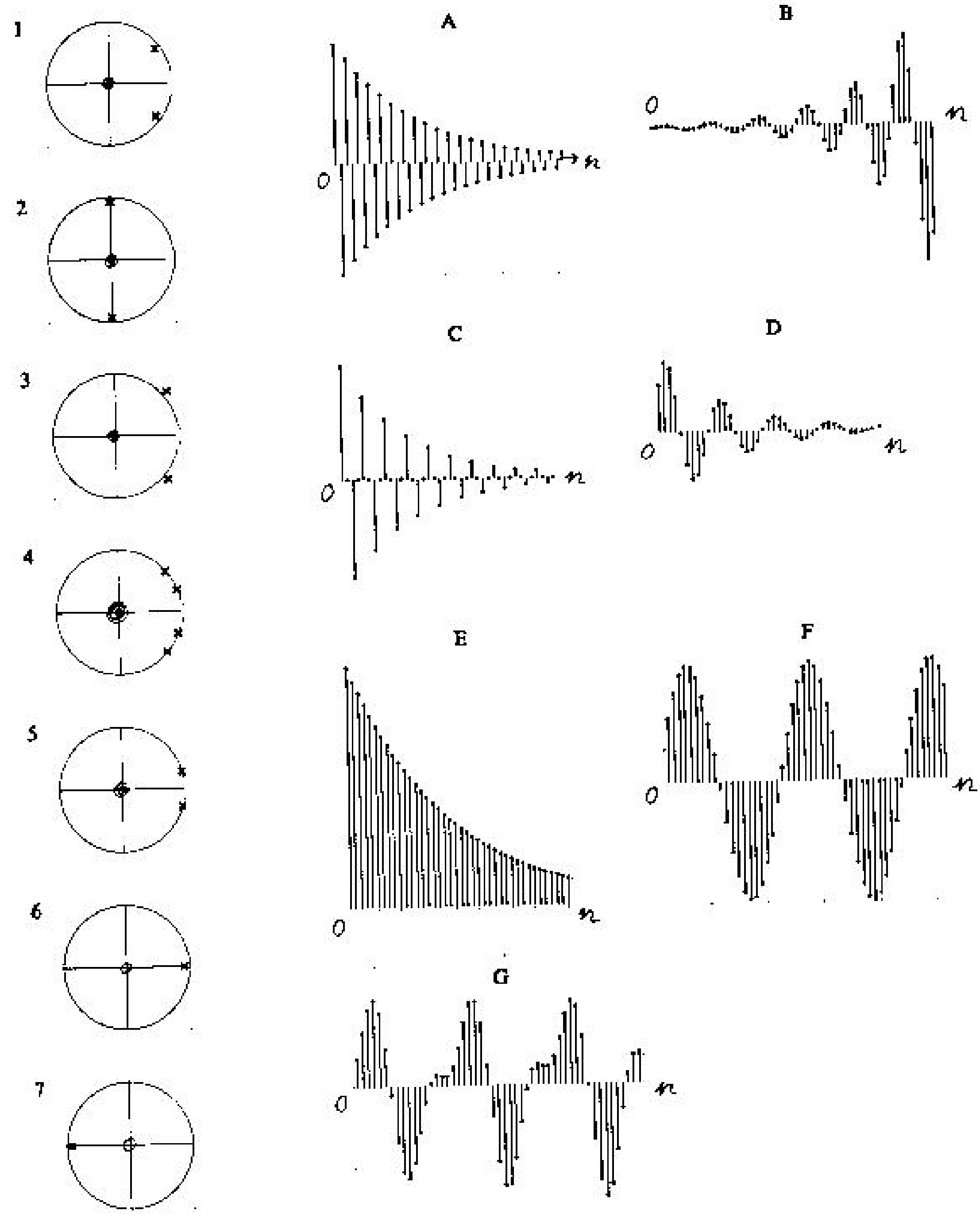
d. Consider

$h[n] = \delta[n] - 2\delta[n-1]$

and L = 3. What is the value of g[0]?
Note: give H[k] first.

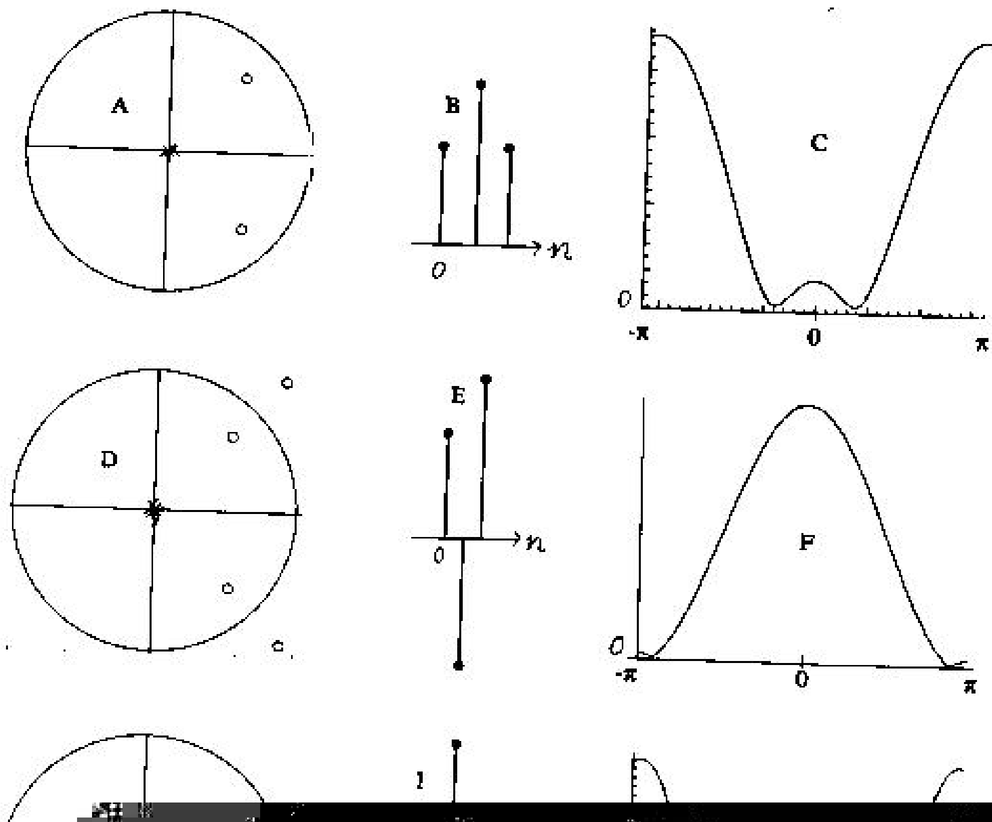
Problem #5a

For the following IIR systems, match the plot with the correct impulse response.
State the reason for the match. (causal systems).


1
2
3
4
5
6
7
A
B
C
D
E
F
G

Problem #5b

For the following FIR systems, match the pole-zero plot with the correct impulse response and frequency response. Sta
twice).



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