Name: ______________________________

UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
and Computer Sciences

Professor David Tse Spring 2000

EECS 121 — MIDTERM

(7:00-9:00 p.m., 8 Wednesday 2000)

Please explain your answers carefully. There are 100 total points and Question 4, part c) is a bonus.

Problem 1 (30 points)

[8 pts.] 1a) Argue that for any binary code satisfying the prefix-free condition, the codeword lengths $\{l_i\}$ must satisfy the Kraft's inequality:

$$\sum_i 2^{-l_i} \le 1 .$$

[6 pts.] b) Is it true that for *any* source, the codeword lengths for the binary Huffman code must satisfy Kraft's inequality with equality? Explain.

[6 pts.] c) Suppose now the coded symbols are from a general alphabet of size D. The Kraft inequality becomes:

$$\sum_i D^{-l_i} \le 1 .$$

Is it true that the Huffman code must satisfy Kraft's inequality with equality? Explain.

[10 pts.] d) Consider a source for which the letter probabilities are of the form 2^{-k}, where k is an integer. Construct the Huffman code and give the corresponding codeword lengths. Justify that the code is optimal.

Problem 2 (30 points)

Let $\{X(t)\}$ be a zero-mean WSS Gaussian process with autocorrelation function $R_x(\tau) = e^{-|\tau|}$.

[6 pts.] **a)** Find its power spectral density.

[8 pts.] **b)** Suppose we sample this process every T seconds. Is the resulting discrete-time process Gaussian? WSS? If so, compute its autocorrelation function.

[8 pts.] **c)** Let $\{Y_n\}$ be the sampled process. We perform DPCM quantization by LLSE prediction of Y_n from Y_{n-1}. Find the distribution of the residual error $Y_n - \hat{Y}_n$.

[8 pts.] **d)** The residual error is quantized by a single bit quantizer to values $\pm\Delta$. Find the optimal choice of Δ as a function of T. What happens when $T \to 0$?

Problem 3 (20 points)

Here is one way to simulate white noise. Let $\{W_n\}$ be iid. rv's with $P(W_n = 1) = P(W_n = -1) = \frac{1}{2}$. For each K, define the continuous process $\{W^{(k)}(t)\}$ for $t \geq 0$ as follows:

$$W^{(k)}(t) = W_n\sqrt{K} \quad \text{for} \quad \frac{n}{K} \leq t \leq \frac{n+1}{K}, \qquad n = 0, 1, 2, \ldots$$

For large K, this can be used to approximate $\{W(t)\}$.

[4 pts.] a) Sketch a typical sample path of $\{W^{(k)}(t)\}$.

[10 pts.] b) Compute $Var[W^{(k)}(t)]$ and $Var\left[\int_0^1 W^{(k)}(t)dt\right]$. Based on this calculation, explain why while $Var[W(t)] = \infty$, $Var\left[\int_0^1 W(t)dt\right]$ is finite.

[6 pts.] c) A student does not like the fact that $Var[W(t)] = \infty$. He decides to use instead the approximation

$$Y^{(k)}(t) = W_n \quad \text{for} \quad \frac{n}{K} \leq t \leq \frac{n+1}{K}, \qquad n = 0, 1, 2$$

What is wrong with this noise model for K large?

Problem 4 (20 points)

A data stream is partitioned into blocks of two bits. A block is modulated onto a signal waveform, say on $[0, 1]$. Consider two modulation schemes:

1) Scheme A: The two bits are modulated into a 4-level PAM, with the 4 levels equally spaced and symmetric about 0.

2) Scheme B: The signal waveform is composed by separately modulating each bit into a 2-level PAM. The waveform for the first bit is on $\left[0, \frac{1}{2}\right]$, and the one for the second bit on $\left[\frac{1}{2}, 1\right]$. The levels are symmetric about 0.

[6 pts.] a) Sketch the possible signal waveforms for both schemes.

[14 pts.] b) Find an orthonormal basis for each scheme (on $[0, 1]$). What are the dimensions of the signal space? Sketch a geometric representation of the signal constellation.

[6 pts.] c) BONUS: An important measure of a modulation scheme is the minimum distance between the constellation points. For a given minimum distance d, find the average energies required in both schemes. Which scheme is better in this respect? (You can assume that each of the four possible messages is equally likely.)