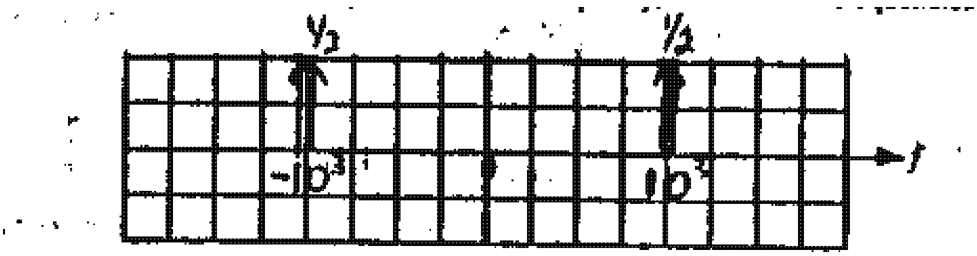
EECS 120 Spring 97 MT2 Solutions-Prof. Fearing

Problem 1.

A)

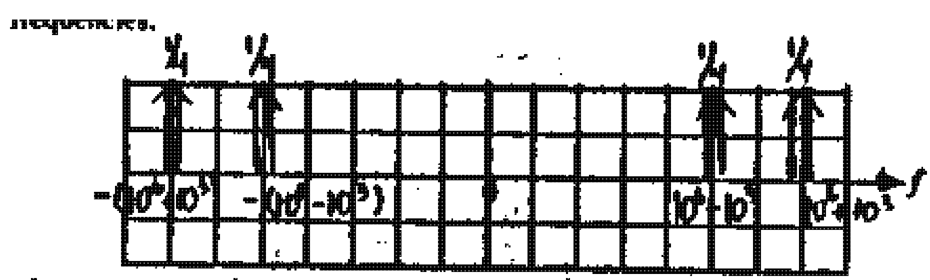


$m(t) = \cos 2000\pi t = \cos(2\pi(1000)t)$

$M(f) = \frac{1}{2}\delta(f-1000) + \frac{1}{2}\delta(f+1000)$

M(f)=(1/2) delta(f-1000)+(1/2)delta(f+1000)

B)



Alternative method

$x_1(t) = (\cos 2\pi 1000t)\cdot(\cos 2\pi f_c t)$

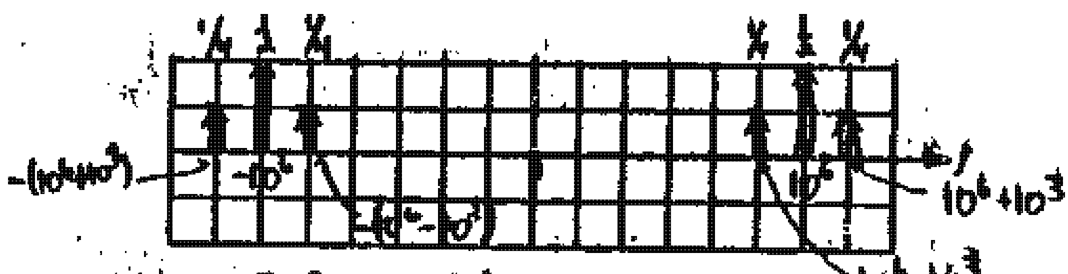
$= \frac{1}{2}\cos(2\pi(f_c-10^3)t) + \frac{1}{2}\cos(2\pi(f_c+10^3)t)$

$X_1(f) = M(f) * \frac{1}{2}[\delta(f-10^6) + \delta(f+10^6)]$

$= \frac{1}{4}\delta(f-10^6-10^3) + \frac{1}{4}\delta(f-10^6+10^3) + \frac{1}{4}\delta(f+10^6-10^3) + \frac{1}{4}\delta(f+10^6+10^3)$

X_1(f)=M(f)=(1/2)[delta(f-10^6+delta(f+10^6)]
=(1/4)delta(f-10^6-10^3)+(1/4)delta(f-10^6+10^3)+(1/4)delta(f+10^6-10^3)+(1/4)delta(f+10^6+10^3)

C)



$x_2(t) = \cos 2\pi f_c t + x_1(t)$

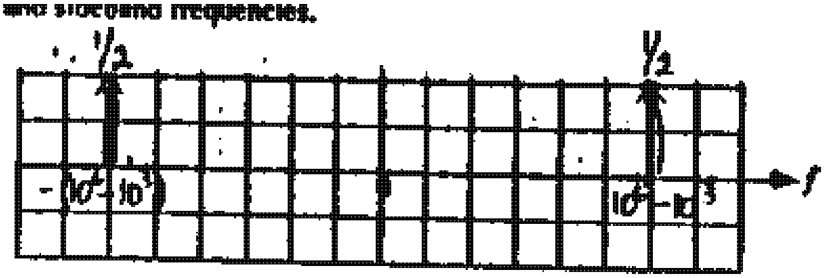
$X_2(f) = \frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c) + X_1(f)$

D)

quency, and sideband frequencies.

$m(t) = \cos 2\pi 1000t$

$n(t) = \sin 2\pi 1000t$

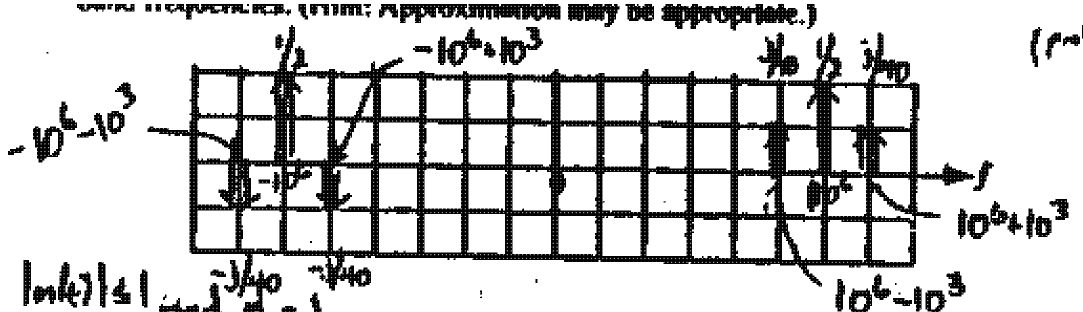


$x_3(t) = \cos 2\pi 10^3 t \cos 2\pi f_c t + \sin 2\pi 10^3 t \sin 2\pi f_c t \overset{\text{by trig identity}}{=} \cos 2\pi(f_c - 10^3)t$

$X_3(f) = \frac{1}{2}\delta(f - (10^6-10^3)) + \frac{1}{2}\delta(f + (10^6-10^3))$

E)

band frequencies. (Hint: Approximation may be appropriate.)



Since $|m(t)| \le 1$ and $\beta = .1$,
Narrow band assumptions can be used

$x_4(t) = \cos 2\pi f_c t - \frac{1}{10} m(t) \sin 2\pi f_c t$

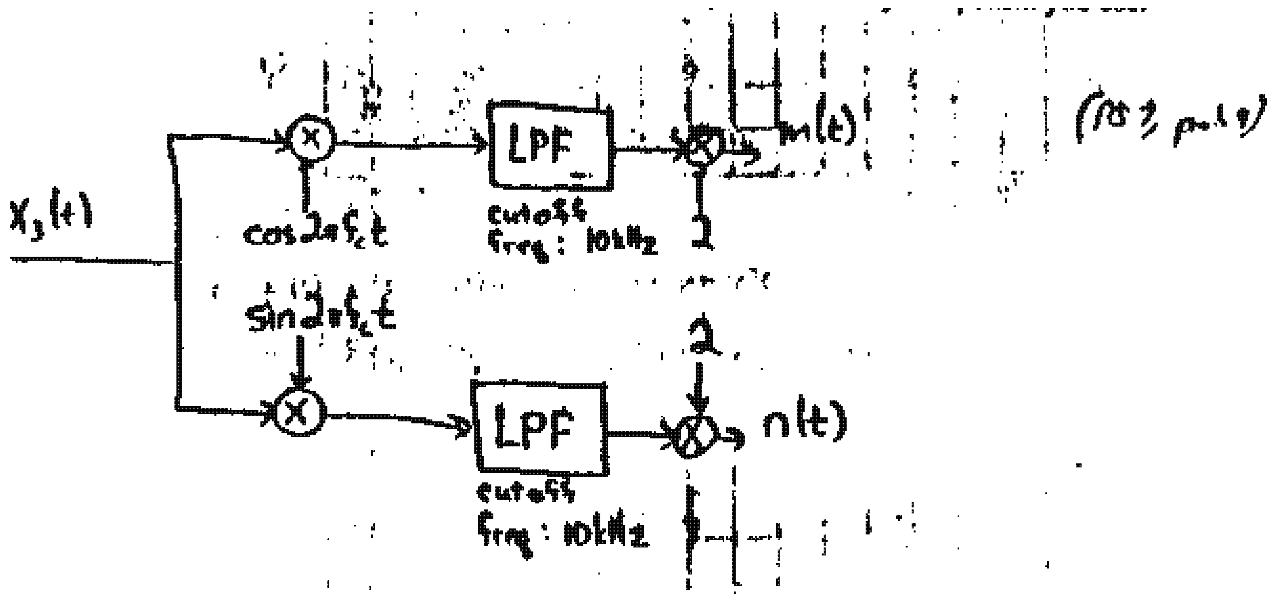
$= \cos 2\pi f_c t - \frac{1}{10}\cos 2\pi 10^3 t \sin 2\pi f_c t$

$= \cos 2\pi f_c t - \frac{1}{20}\left[\sin 2\pi(f_c - 10^3)t + \sin 2\pi(f_c+10^3)t\right]$

$X_4(f) = \frac{1}{2}[\delta(f-f_c) + \delta(f+f_c)] - \frac{j}{40}\Big[\delta(f+(f_c-10^3)) - \delta(f-(f_c-10^3)) + \delta(f+(f_c+10^3)) - \delta(f-(f_c+10^3))\Big]$

Note: All these problems can also be easily done graphically

F)



G)

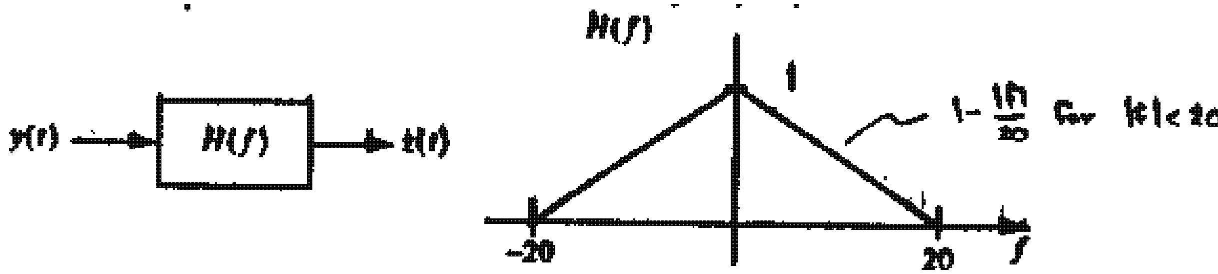
$x_1(t)$ modulation type AM DSB

$x_2(t)$ modulation type AM DSB LC

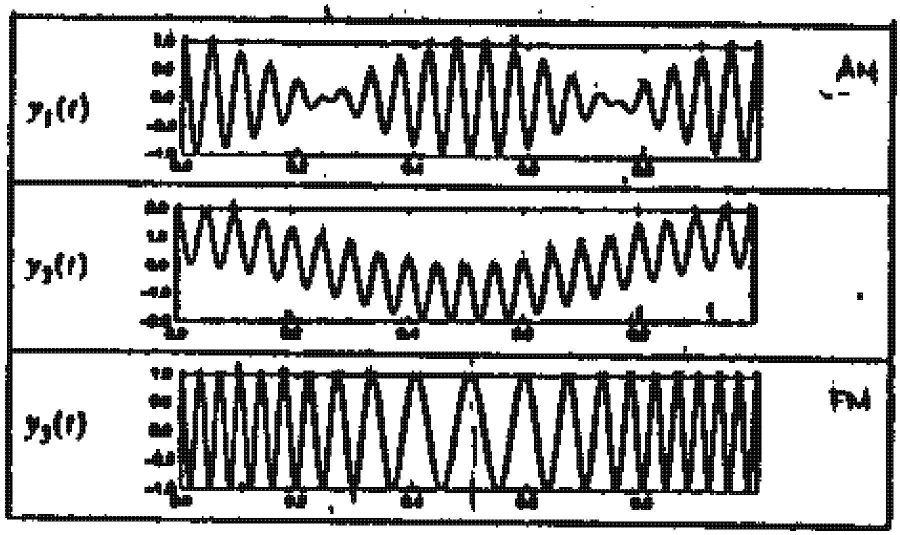
$x_3(t)$ modulation type QAM

$x_4(t)$ modulation type NBPM

Problem 2



Note: All $y_1(t)$, $y_2(t)$, $y_3(t)$ are periodic. The horizontal axis has units of seconds.



$y_1(t) = \cos 2\pi t \cos 2\pi(20)t$

$= \frac{1}{2}\cos 2\pi(19)t + \frac{1}{2}\cos 2\pi(21)t$

$y_2(t) = \cos 2\pi t + \cos 2\pi($

$y_3(t) = \cos(2\pi 20t + 2\pi \sin 2\pi t$

A)

What is $z_1(t)$, the output of the filter for input $y_1(t)$?

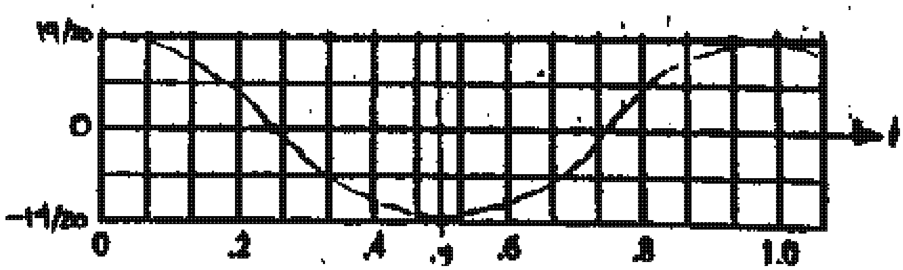
$z_1(t)$ is $\frac{1}{40}\cos 2\pi(19)t$

by $\frac{1}{20}$

B)

Sketch $z_2(t)$, the output of the filter for input $y_2(t)$.

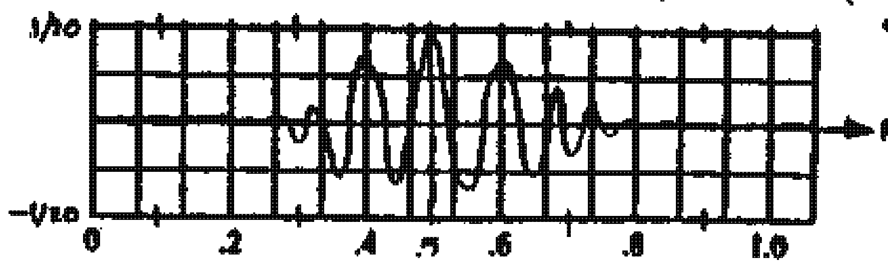
by $\frac{1}{20}$



C)

e) BONUS: Sketch $z_3(t)$, the output of the filter for input $y_3(t)$.

(Similar to discriminator discussed in lecture)

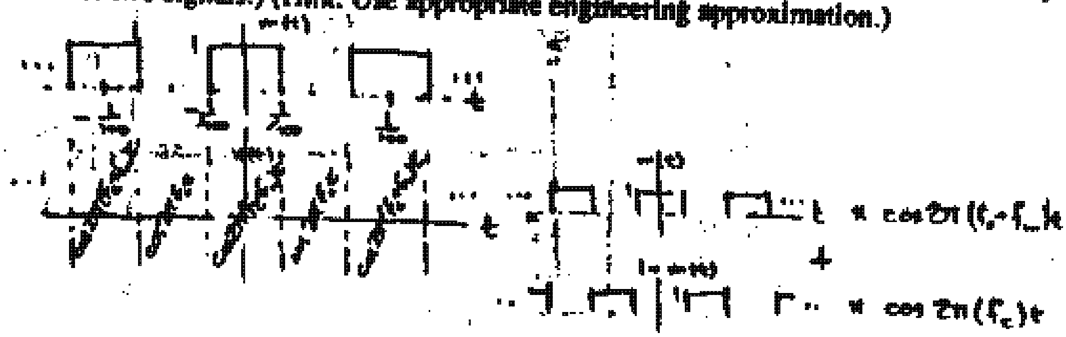


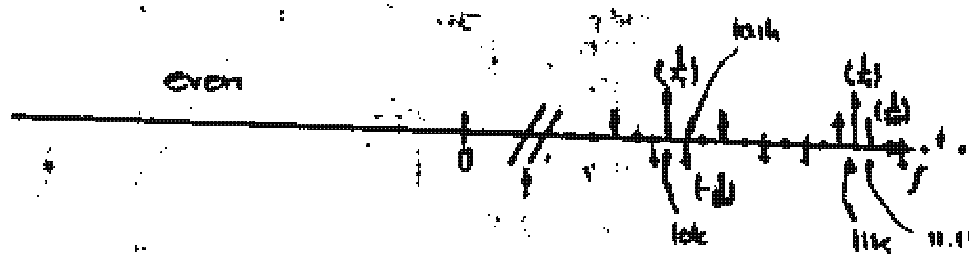
lowest instantaneous frequency at t=.5 gets least attenuated

Problem 3

A)

of two signals.) (Hint: Use appropriate engineering approximation.)

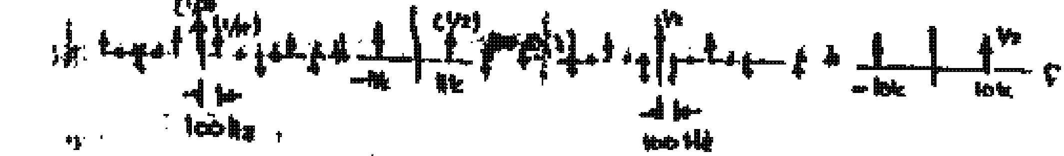




Engineering approximation: distant overlapping spikes do not greatly affect each other's magnitude

since $x(t) = m(t)\cos 2\pi(f_c+f_m)t + [1-m(t)]\cos 2\pi f_c t$

so $X(f) = M(f) * \mathcal{F}[\cos 2\pi(f_c+f_m)t] + [\delta(f)-M(f)] * \mathcal{F}[\cos 2\pi f_c t]$



B)

What is the power in $x(t)$? $\underline{\quad \frac{1}{2} \quad}$

the power in a sinusoid with amplitude A is $\frac{A^2}{2}$, regardless of frequency and phase.

$\int |A\cos(2\pi f_c t + \phi)|^2 dt = \frac{1}{2}$

What fraction of the power in $X(f)$ is at the carrier frequency f_c? $\boxed{\frac{1/8}{1/2} = \frac{1}{4}}$

power at f_c = tone is $2\left|\frac{1}{4}\right|^2 = \frac{1}{8}$

Problem 4

A)

What is the minimum number of poles the system must have? $\underline{\quad 3 \quad}$

so after completing (b), note that $|H(\omega)| \sim \frac{1}{\omega}$

3 poles gives the desired rolloff

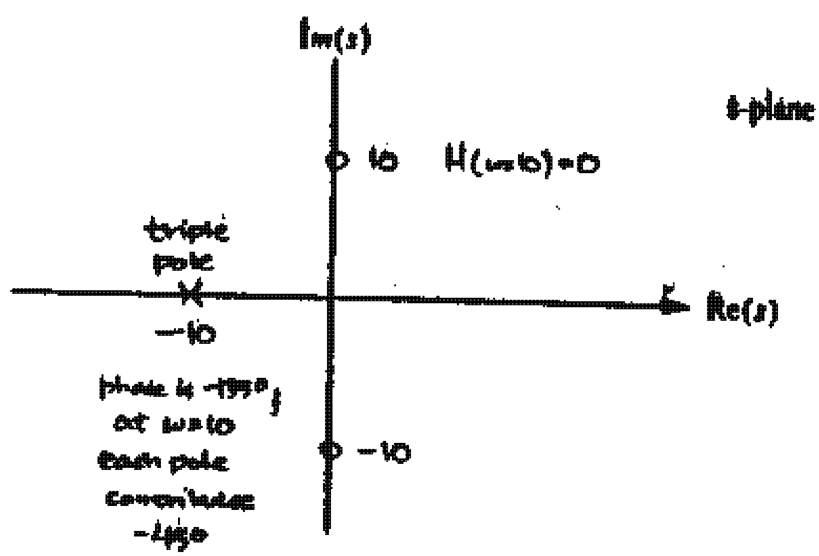
B)

What is the minimum number of zeros the system must have? $\underline{\quad 2 \quad}$

system must have 2 zeros; $H(\omega=10)=0$ and system is real so poles and zeros must appear in complex conjugate pairs

C)

Sketch and label the pole-zero diagram for a stable system (using a minimum number of poles and zeros) which would have the given magnitude response. Note: $H(\omega = 0) = 0.1$, $H(\omega = 10) = 0$.

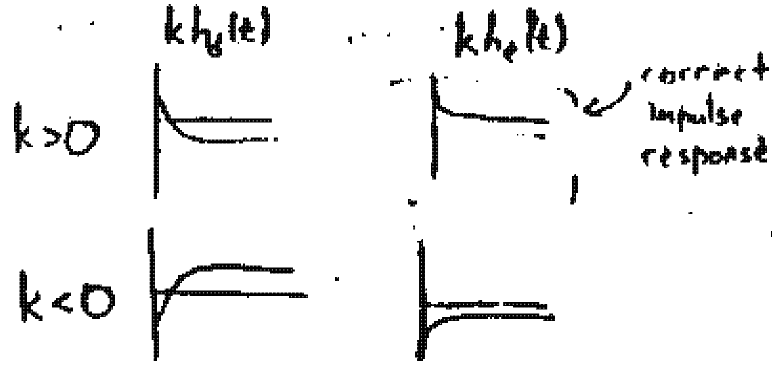


Problem 5

Impulse response	Matching pole-zero plot	Impulse response	Matching pole-zero plot
$h_1(t)$	E	$h_2(t)$	F
$h_3(t)$	G	$h_4(t)$	D
$h_5(t)$	$\boxtimes$	$h_6(t)$ Bonus 3 pts.	C

none of A...I. bug in impulse response plot.

$h_1(t)$: decays to final value of 1 $\Rightarrow$ pole at origin
possible solutions: D or E

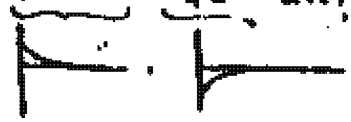


Answer: E

$h_2(t)$: decays to zero $\Rightarrow$ all poles in LHP
no oscillations $\Rightarrow$ all poles on real axis
possible solutions: C or F

Answer: F

$h_C(t) = \underbrace{\tfrac{1}{2}e^{-t}u(t)}\ -\ \underbrace{\tfrac{1}{2}e^{-10t}u(t)}$



$h_F(t) = \tfrac{1}{3}e^{-t}u(t) + \tfrac{4}{3}e^{-10t}u(t)$

$h_3(t)$: oscillations $\Rightarrow$ complex conjugate poles
possible solutions: G or I or B
oscillations increase in magnitude with time $\Rightarrow$ poles in RHP

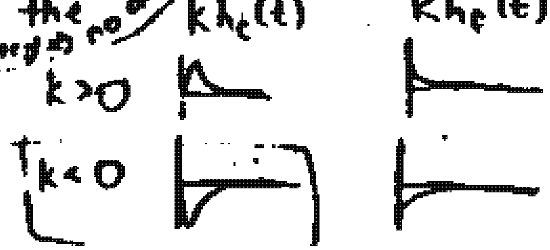
Solution: G

$h_4(t)$: decays to non-zero value $\Rightarrow$ pole at origin
possible solutions: D or E
looking at graphs above $\Rightarrow$ answer: D

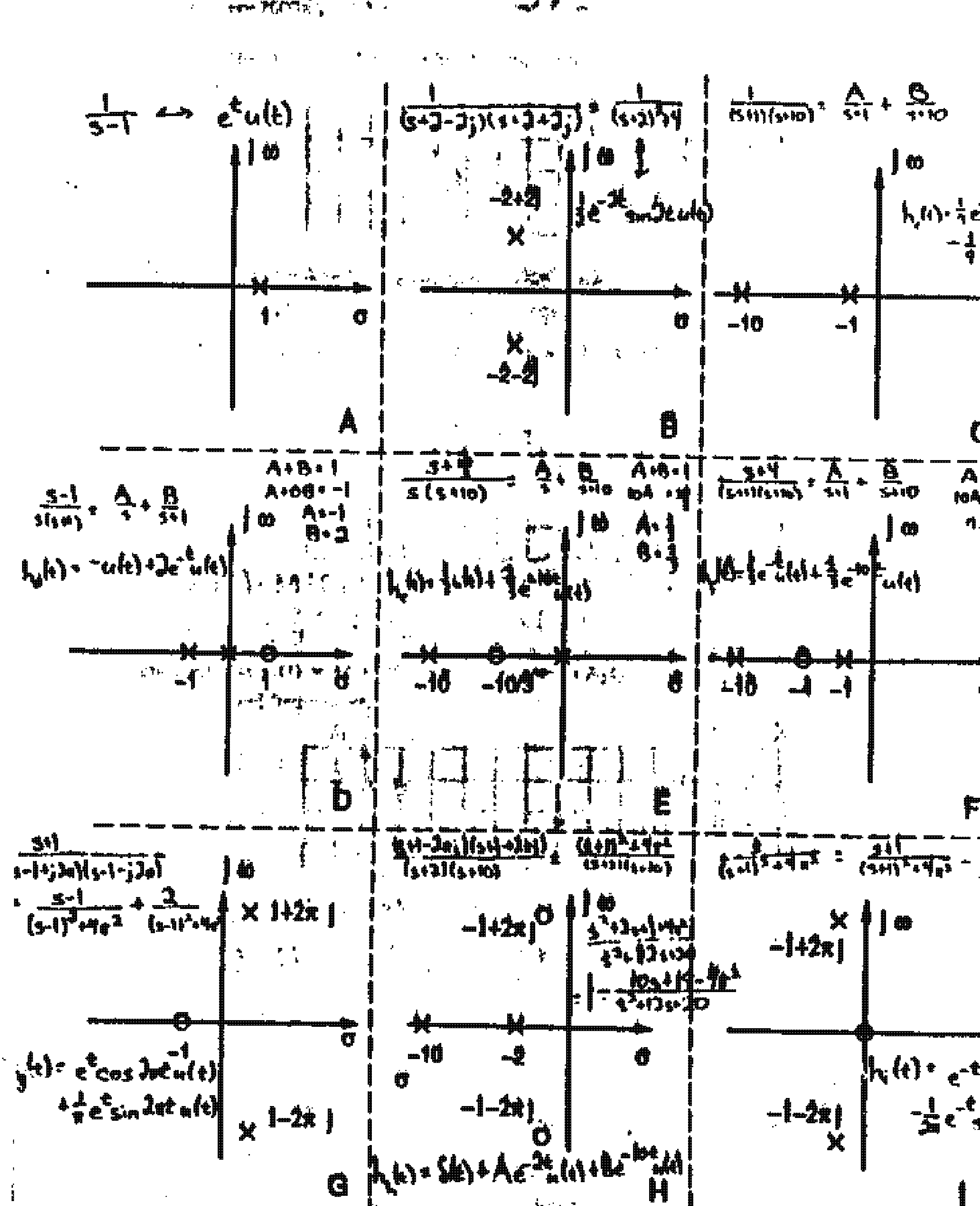
$h_5(t)$: oscillations $\Rightarrow$ complex conjugate poles
possible solutions: G, B, or I
oscillations decrease in magnitude with time $\Rightarrow$ poles in LHP
possible solutions: B or I
$h_5(0)=0$ $\Rightarrow$ no cosine component
possible answer: B, but the sin is at the wrong frequency $\Rightarrow$ no answer. Everybody gets credit.

$h_6(t)$: decays to zero $\Rightarrow$ all poles in LHP
no oscillations $\Rightarrow$ all poles on real axis
possible solutions: C or F

answer: C



These pole–zero diagrams are possible answers for the questions of Problem 5. All diagrams represent causal systems.



Problem 6

A)

With $d(t) = 0$, compute $\frac{Y(s)}{X(s)} = \boxed{\frac{4ks}{s^2+4ks+4}}$

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{4ks/s^2+4}{1+4ks/s^2+4} = \frac{4ks}{s^2+4ks+4}$$

B)

b) For which values of k is the system stable?

k: $k>0$

poles at

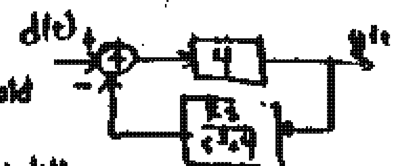
$$s = \frac{-4k \pm \sqrt{(4k)^2-16}}{2}$$

for stability, no poles in RHP or $j\omega$ axis (except for a single pole at the origin)

C)

$\lim_{t\to\infty} y(t) = \boxed{0}$

(If limit exists, answer should be a number) otherwise, write 'does not exist'



$D(s) = \frac{2}{s^2+4}$

$Y(s) = \frac{4}{(s+2)^2}$

$\lim_{s\to 0} sY(s) = 0$

$$\frac{Y(s)}{D(s)} = \frac{4(s^2+4)}{\underbrace{s^2+4s+4}_{}}$$

poles at $s=-2,-2$

D)

[4 pts.] d) Let $d(t) = 0$ and $x(t) = tu(t)$, with $k = 1$.

$$\lim_{t \to \infty} y(t) = \boxed{1}$$

(If limit exists, answer should be a number)
otherwise, write 'does not exist'

$$Y(s) = \frac{4s}{s^2+4s+4} X(s)$$

$$x(t) = \int_0^t u(t)\,dt$$

$$X(s) = \frac{1}{s}\left(\frac{1}{s}\right) = \frac{1}{s^2}$$

$$Y(s) = \frac{4}{s(s^2+4s+4)}$$

$Y(s)$ has no poles in the RHP so Final Value Theorem is applicable

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} \frac{4}{s^2+4s+4}$$

$$= 1$$