1. **15 points** Find the FT of *x*, and sketch the real and imaginary parts of X(ω), where

$$\forall t, x(t) = \prod(t) * \prod(t) * \sum_{-\infty}^{\infty} \delta(t - 8n)$$

Here $\prod(t) = 1\ for\ |t| \leq \frac{1}{2}\ and\ 0, else$. In your sketch carefully mark the relevant frequencies and magnitudes.

2. **15 points**

(a) Find and sketch the FT of

$$x(t) = \left[\frac{\sin \pi t}{\pi t}\right]^2 e^{-\,j2\pi\, x\, 10t}.$$

(b) Use Parseval's theorem to find the energy in the signal x, $\int_{-\infty}^{\infty} [x(t)]^2 dt$.

3. **10 points** The following statements are either TRUE or FALSE. If you believe a statement is true, outline a BRIEF PROOF. If you believe it is false, provide a BRIEF COUNTEREXAMPLE.

(a) If $x(t), t \in Reals$, is a real valued signal, its Fourier transform $X(f), f \in Reals$, is also real valued.

(b) If $x(t), y(t), t \in Reals$, are real-valued signals and $(x * y)(t) = 0, \forall t \in Reals$, then either x or y is identically zero.

(c) If $x(t), t \in Reals$, is a real-valued, baseband signal with bandwidth W Hz, then the signal $y, y(t) = 2x^2(t) + 3x^4(t), t \in Reals$, has bandwidth at most $4W$Hz.

(d) If $x(t), t \in Reals$ is a real-valued, band-limited signal with bandwidth W Hz, then the signal $y(t) = x, t \in Reals$, has a bandwidth W^2 Hz.

(e) If x, y are real-valued signals with bandwidth W_x, W_y Hz, respectively, then a the signal $x + y$ has bandwidth $W_x + W_y$ Hz.

4. **15 points** The following statements are either TRUE or FALSE. If you believe a statement is true, outline a BRIEF PROOF. If you believe it is false, provide a BRIEF COUNTEREXAMPLE.

   (a) The system that takes as input a signal m and produces its Hilbert transform $\widehat{m}$ as output is an LTI system.

   (b) The SSB-USB modulator which takes as input a signal $m(t), t \in Reals$, and produces as output the modulated signal $x(t), t \in Reals$, is a linear system.

   (c) The narrow-band FM system which takes as input the continuous-time signal m and produces as output the modulated signal x, is a linear system.

   (d) The AM-DSB modulator is a time-invariant system.

   (e) The signal $\forall t, x(t) = \cos(2\pi f_c t + \cos(2\pi f_m t))$ $(f_m \neq 0)$ has infinite bandwidth.

   (f) Is it possible to recover the signals A and θ from the narrowband signal $\forall t, x(t) = A(t)\cos\big(2\pi f_c t + \theta(t)\big)$.

5. **20 points** Figure 1 is a block diagram of vestigial sideband (VSB) modulation/demodulation.

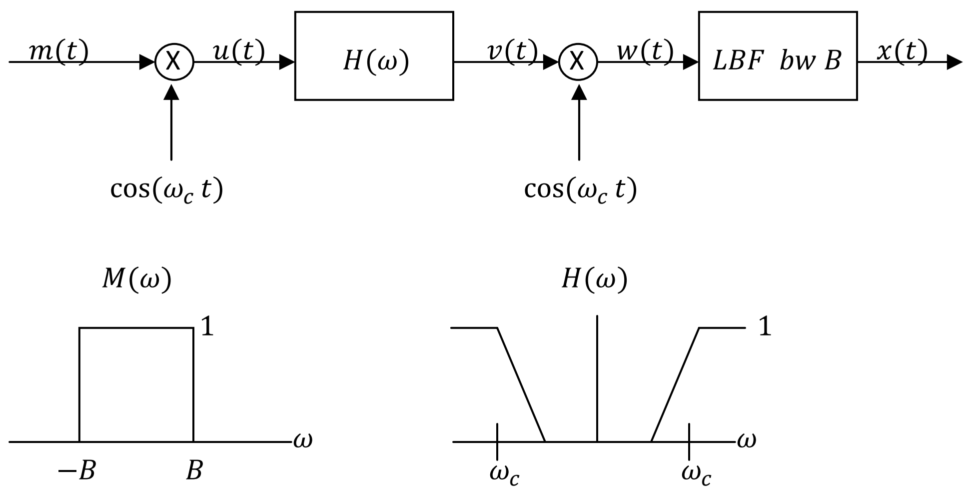

Figure 1: The VSB modulation-demodulation scheme of problem 5

The baseband signal m has FT M as shown, with bandwidth B rad/sec. It modulates the carrier $\cos(\omega_c t)$ ($\omega_c \gg B$) to produce the signal u, which is passed through the VSB filter, whose frequency response $H(\omega)$ is shown. The result is the transmitted signal v. The coherent receiver multiplies v by the carrier to produce w, which is then passed through a low pass filter (LPF) (with bandwidth B) to obtain the signal x.

(a) Obtain mathematical expressions for the FT of u, v, and w. Sketch the FTs, and carefully mark the relevant magnitudes and frequencies.

(b) Show that $x = (1/4)m$ if the VSB filter satisfies

$$H(\omega + \omega_c) + H(\omega - \omega_c) = 1, for\ |\omega| \leq B.$$

6. **10 points** Figure 2 is a block diagram of a digital communication system. The digital channel accepts

Figure 2: The communication system for problem 6

at its input port any symbol from $\{a, b, c, d, e\}$ and delivers it at its output port. The channel can accept one symbol every 2μ sec.

(a) What is the baud rate of the channel in symbols/sec? What is its capacity in bits/sec?

(b) A binary source m produces data at 1 Mb/sec. (1 Mb is one million bits.) Is the rate of the source smaller than the capacity? If it is, construct a "coder" that maps the binary source m into a sequence of symbols x, and a "decoder" that maps x into a binary sequence m' such that $m' = m$.

7. **15 points** m is a **complex-valued** signal with bandwidth B_m rad/sec whose real and imaginary parts are m_1, m_2 respectively. Let $M(\omega), M_1(\omega)\ and\ M_2(\omega)$ be the FT of $m, m_1, and\ m_2$ respectively.

(a) Find M_1 and M_2 in terms of M. Show that the bandwidth of m_1, m_2 is at most B_m.

(b) Design a modulation and demodulation scheme that can transmit m_1 and m_2 over a channel with bandwidth $2B_m$ centered at frequency ω_c rad/sec.

(c) Give a brief mathematical argument to show that the transmitted signal is within the channel bandwidth, and that the receiver can recover both signals.