Problem 1 (Amplitude Modulation.) 40 Points

(a) (10 Pts) For the discrete-time signal x[n] it is known that X(e^(jw)) = 0, for |w| > pi/4. Determine the range of w for which the DTFT of y[n] = cos(5*pi*n/4)x[n] must be zero. Hint: Select an example spectrum X(e^(jw)) and sketch the resulting DTFT of y[n].
(b) (10 Pts) The real-valued data signal x(t) is known to be band-limited, i.e., X(jw) = 0, for |w| > W. Consider the block diagram of Figure 1, where

```
H_1(jw) = {1, for |w| <= w_c   and   H_2(jw) = {1, for |w| >= 2w_c    (3)
          {0  otherwise,                        {0  otherwise.
```

- Pick an arbitrary (bandlimited) example spectrum for x(t), and sketch the corresponding spectrum of the signal y(t).
- For what values of the parameters W and w_c is it possible to recover x(t) from y(t)?
- Provide the block diagram of a system that recovers x(t), given y(t), carefully specifying all involved paramters.

```
                 cos(w_c*t)
                     |
                     |
                     V
              ------> X ------> H_1(jw)-----
              |                            |
              |                            V
     ___x(t)_|                            + -------> y(t)
              |                            ^
              |                            |
              |                            |
              ------> X ------> H_2(jw)-----
                     ^
                     |
                     |
                 cos(2W_c*t)
```

Figure 1: Block diagram for Part (b).

(c) (10 Pts) The real-valued data signal x(t) is known to be band-limited, i.e., X(jw)=0, for |w|>W. The goal is to perform standard (i.e., double-sideband) AM with carrier frequency w_c>5W. Unfortunately, the only type of modulator available is multiplication by cos(w_c*t/4). Otherwise, addition, scalar multiplication, and filters can be used. Draw the block diagram of the system that achieves our goal, and if your system uses a filter, specify the desired frequency respones. Hint: Pick an example spectrum for x(t) and sketch the spectra of intermediate signals to maximize your chances for partial credit.
(d) (10 Pts) The real-valued data signal x(t) is known to be band-limited, i.e., X(jw)=0, for |w|>W. The goal is to perform single-sideband AM with only the lower sideband, with carrier frequency w_c>5W. Again, you can use addition, scalar multiplication, and multiplication by cos(w_m*t), for arbitrary w_m. However, this time, you only have fixed ideal low-pass filters with the folowing frequency response:

```
H(jw) = {1, for |w| <= w_c/2
        {0  otherwise.
```

Draw the block diagram of a system that achieves the goal, clearly specifying all involved parameters, such as the frequencies of the modulators, etc. Hint: Pick an example spectrum for x(t) and sketch the spectra of intermediate signals to maximize your changes for partial credit.

Problem 2(PAM.) 30 Points

Two pulses are suggested for a PAM system:

p_1(t) = ae^(-t)*u(t), and p_2(t) = be^(-10t)*u(t),

where a and b are positive real numbers that will be selected appropriately, leading to

y_i(t) = sum from k= -infinity to +infinity x[k]p_i(t-kT), for i = 1,2,

where we choose T=1. We suppose that the data signal is bounded to |x(t)|<=1. In this problem, we want to compare the two pulses p_1(t) and p_2(t).
(a) (20 Pts) Select a=2 and b=2(10)^(1/2), For this choice, it can be shown that the pulse energy is the same for p_1(t) and p_2(t). (You don't have to show this!) Now consider the transmission of p_1(t) and p_2(t), respectively, across a communication channel with impulse response h(t) and corresponding frequency response

H(jw) = 1/(6 + jw).

This yields an output signal z_i(t) = (p_i * h)(t), for i = 1,2.
- Evaluate the energy of the received signals, z_1(t) and z_2(t), respectively.
- Which received signal has the larger energy?
- How is it possible that even though the two pulses have the same transmitted energy, their received energies differ?
(b) (10 Pts) (Hard problem) To have a fair comparison, we have to make sure that hte powers of the transmitted signals y_1(t) and y_2(t),respectively, are equal. To adjust the power, assume that x[n]=1 for all n, i.e., for -infinity < n < +infinity. Determine the relationship between a and b such that for this particular x[n], the signals y_1(t) and y_2(t) have the same power. (As seen in class, this provides a worst case analysis.) Hint: By contrast to Part (a), this question studies the power of the entire signal, rather than the energy of a single pulse.

Problem 3 (Sampling.) 30 Points

A real-valued data signal x(t) is known to be band-limited, i.e., X(jw)=0, for |w|>W.
(a) (14 Pts) Suppose the signal x(t) is sampled non-uniformly using the impulse train q_1(t) shown in Figure 2. Show that the spectrum of the sampled signal y_1(t) = x(t)q_1(t) is

Y_1(jw) = (1/2T) sum from k= -infinity to +infinity (1+e^(-j*pi*k/4))X(j(w-pi*k/T)).

Carefully justify every step in your derivation, including references to results from the tables.

```
                                         ^ q_1(t)
                                         |
                                         |
             ^   ^                       ^   ^                       ^   ^
             |   |                       |   |                       |   |
             |   |                       |   |                       |   |
             |   |                       |   |                       |   |
     ...     |   |                       |   |                       |   |
_____________|__|____________________|__|____________________|__|___________
          -2T   -7T/4                    T/4                  2T   9T/4
```

Figure 2: The sampling impulse train q_1(t), where T = pi/W.

(b) (5 Pts) Is it possible to low-pass filter the signal y_1(t) to get back the signal x(t)?
Answer: yes/no. (circle one)
Explanation: (Hint: Pick an example band-limited spectrum for x(t), and sketch the resulting spectrum of y_1(t). Based on your plot, explain.)
(c) (5 Pts) Suppose the signal is sampled non-uniformly using the impulse train q_2(t) shown in Figure 3. The impulses are in teh same locations as in Figure 2, but they have different weights a and b (both real numbers). Find an expression for the spectrum Y_2(jw) of the sampled signal y_2(t) = x(t)q_2(t).

```
                                         ^ q_2(t)
                                         |
                                         |
             ^a                          ^a                          ^a
             |                           |                           |
             |  ^b                       |  ^b                       |  ^b
             |   |                       |   |                       |   |
     ...     |   |                       |   |                       |   |
_____________|__|____________________|__|____________________|__|___________
          -2T   -7T/4                    T/4                  2T   9T/4
```

Figure 3: The sampling impulse train q_2(t), where T=pi/W.

(d) (5 Pts) For |w|

```
                    Y(jw) = {Y_1(jw)+jY_2(jw), if w>=0,
                    Select the real numbers a and b such that for 0 < w < W
                            {Y_1(jw)-jY_2(jw), if w
 (e) (1 Pt) It can also be shown that with the choice of a and b as in Part (d)
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