

### Problem 1 (Amplitude Modulation.) 40 Points

(a) (10 Pts) For the discrete-time signal  $x[n]$  it is known that  $X(e^{j\omega}) = 0$ , for  $|\omega| > \pi/4$ . Determine the range of  $\omega$  for which the DTFT of  $y[n] = \cos(5\pi n/4)x[n]$  must be zero. Hint: Select an example spectrum  $X(e^{j\omega})$  and sketch the resulting DTFT of  $y[n]$ .

(b) (10 Pts) The real-valued data signal  $x(t)$  is known to be band-limited, i.e.,  $X(j\omega) = 0$ , for  $|\omega| > W$ . Consider the block diagram of Figure 1, where

$$H_1(j\omega) = \begin{cases} 1, & \text{for } |\omega| \leq W \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad H_2(j\omega) = \begin{cases} 1, & \text{for } |\omega| \geq 2W_c \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

- Pick an arbitrary (bandlimited) example spectrum for  $x(t)$ , and sketch the corresponding spectrum of the signal  $y(t)$ .
- For what values of the parameters  $W$  and  $\omega_c$  is it possible to recover  $x(t)$  from  $y(t)$ ?
- Provide the block diagram of a system that recovers  $x(t)$ , given  $y(t)$ , carefully specifying all involved parameters.

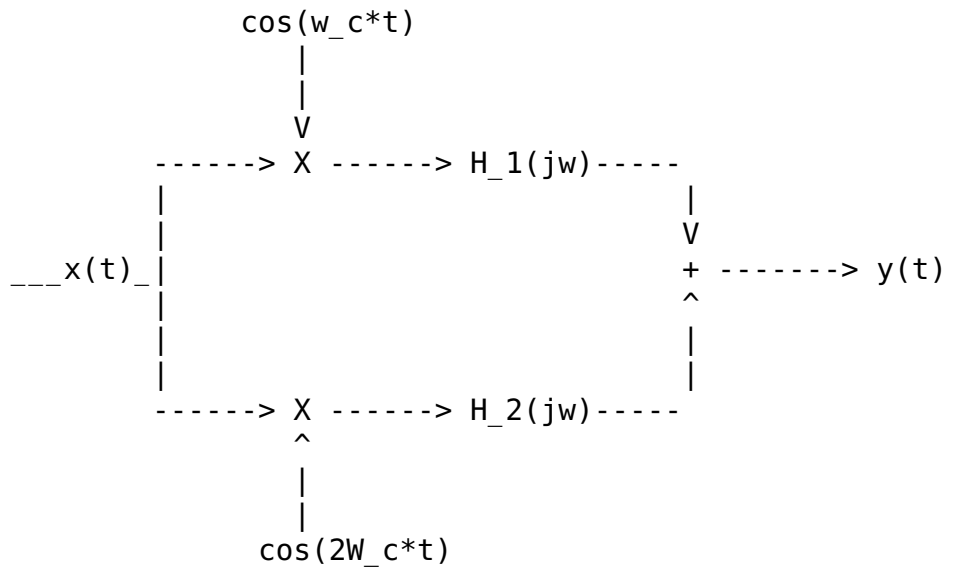


Figure 1: Block diagram for Part (b).

(c) (10 Pts) The real-valued data signal  $x(t)$  is known to be band-limited, i.e.,  $X(j\omega)=0$ , for  $|\omega|>W$ . The goal is to perform standard (i.e., double-sideband) AM with carrier frequency  $\omega_c>5W$ . Unfortunately, the only type of modulator available is multiplication by  $\cos(\omega_c t/4)$ . Otherwise, addition, scalar multiplication, and filters can be used. Draw the block diagram of the system that achieves our goal, and if your system uses a filter, specify the desired frequency responses. Hint: Pick an example spectrum for  $x(t)$  and sketch the spectra of intermediate signals to maximize your chances for partial credit.

(d) (10 Pts) The real-valued data signal  $x(t)$  is known to be band-limited, i.e.,  $X(j\omega)=0$ , for  $|\omega|>W$ . The goal is to perform single-sideband AM with only the lower sideband, with carrier frequency  $\omega_c>5W$ . Again, you can use addition, scalar multiplication, and multiplication by  $\cos(\omega_m t)$ , for arbitrary  $\omega_m$ . However, this time, you only have fixed ideal low-pass filters with the following frequency response:

$$H(j\omega) = \begin{cases} 1, & \text{for } |\omega| \leq \omega_c/2 \\ 0 & \text{otherwise.} \end{cases}$$

Draw the block diagram of a system that achieves the goal, clearly specifying all involved parameters, such as the frequencies of the modulators, etc. Hint: Pick an example spectrum for  $x(t)$  and sketch the spectra of intermediate signals to maximize your changes for partial credit.

## Problem 2(PAM.) 30 Points

Two pulses are suggested for a PAM system:

$$p_1(t) = ae^{-(t)}u(t), \text{ and } p_2(t) = be^{(-10t)}u(t),$$

where  $a$  and  $b$  are positive real numbers that will be selected appropriately, leading to

$$y_i(t) = \sum_{k=-\infty}^{+\infty} x[k]p_i(t-kT), \text{ for } i = 1,2,$$

where we choose  $T=1$ . We suppose that the data signal is bounded to  $|x(t)| \leq 1$ . In this problem, we want to compare the two pulses  $p_1(t)$  and  $p_2(t)$ .

(a) (20 Pts) Select  $a=2$  and  $b=2(10)^{(1/2)}$ , For this choice, it can be shown that the pulse energy is the same for  $p_1(t)$  and  $p_2(t)$ . (You don't have to show this!) Now consider the transmission of  $p_1(t)$  and  $p_2(t)$ , respectively, across a communication channel with impulse response  $h(t)$  and corresponding frequency response

$$H(j\omega) = 1/(6 + j\omega).$$

This yields an output signal  $z_i(t) = (p_i * h)(t)$ , for  $i = 1,2$ .

- Evaluate the energy of the received signals,  $z_1(t)$  and  $z_2(t)$ , respectively.

- Which received signal has the larger energy?

- How is it possible that even though the two pulses have the same transmitted energy, their received energies differ?

(b) (10 Pts) (Hard problem) To have a fair comparison, we have to make sure that the powers of the transmitted signals  $y_1(t)$  and  $y_2(t)$ , respectively, are equal. To adjust the power, assume that  $x[n]=1$  for all  $n$ , i.e., for  $-\infty < n < +\infty$ . Determine the relationship between  $a$  and  $b$  such that for this particular  $x[n]$ , the signals  $y_1(t)$  and  $y_2(t)$  have the same power. (As seen in class, this provides a worst case analysis.)

Hint: By contrast to Part (a), this question studies the power of the entire signal, rather than the energy of a single pulse.

## Problem 3 (Sampling.) 30 Points

A real-valued data signal  $x(t)$  is known to be band-limited, i.e.,  $X(j\omega)=0$ , for  $|\omega| > W$ .

(a) (14 Pts) Suppose the signal  $x(t)$  is sampled non-uniformly using the impulse train  $q_1(t)$  shown in Figure 2. Show that the spectrum of the sampled signal  $y_1(t) = x(t)q_1(t)$  is

$$Y_1(j\omega) = (1/2T) \sum_{k=-\infty}^{+\infty} (1 + e^{-j\pi k/4}) X(j(\omega - \pi k/T)).$$

Carefully justify every step in your derivation, including references to results from the tables.

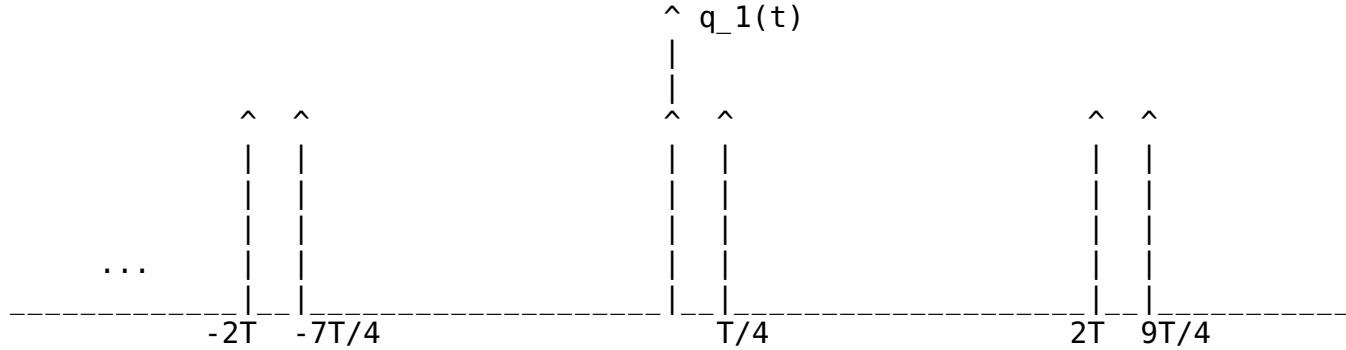


Figure 2: The sampling impulse train  $\hat{q}_1(t)$ , where  $T = \pi/W$ .

(b) (5 Pts) Is it possible to low-pass filter the signal  $y_1(t)$  to get back the signal  $x(t)$ ?

Answer: yes/no. (circle one)

Explanation: (Hint: Pick an example band-limited spectrum for  $x(t)$ , and sketch the resulting spectrum of  $y_1(t)$ . Based on your plot, explain.)

(c) (5 Pts) Suppose the signal is sampled non-uniformly using the impulse train  $\hat{q}_2(t)$  shown in Figure 3. The impulses are in the same locations as in Figure 2, but they have different weights  $a$  and  $b$  (both real numbers). Find an expression for the spectrum  $Y_2(j\omega)$  of the sampled signal  $y_2(t) = x(t)\hat{q}_2(t)$ .

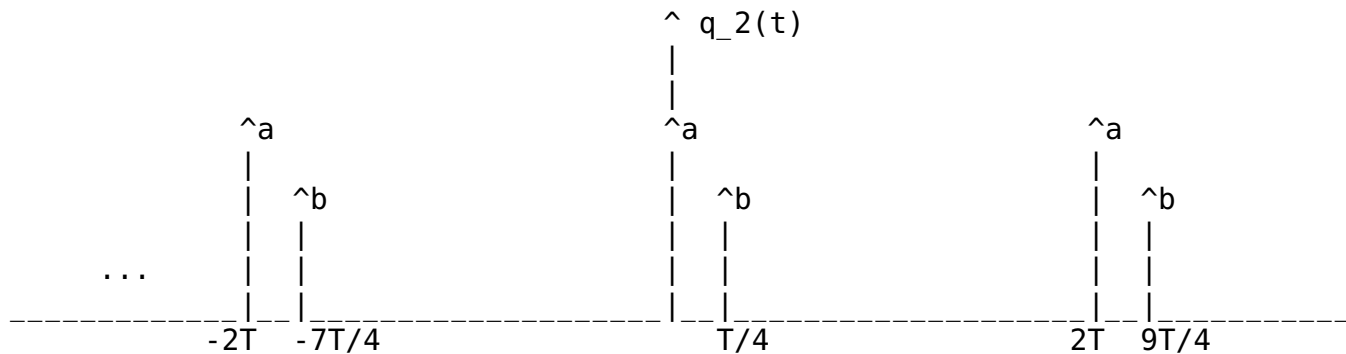


Figure 3: The sampling impulse train  $\hat{q}_2(t)$ , where  $T = \pi/W$ .

(d) (5 Pts) For  $\omega$

$$Y(j\omega) = \{Y_1(j\omega) + jY_2(j\omega), \text{ if } \omega \geq 0,$$

Select the real number  $\omega_c$  and  $\omega$  such that for  $0 < \omega < \omega_c$

(e) (1 Pt) It can also be shown that with the choice of  $a$  and  $b$  as in Part (d)