EECS 120
Midterm 1
Wed. Oct. 26, 2016: 1610 - 1800 pm

Name:____________________________
SID:______________________

For statistical purposes only:
Circle courses you have taken EE20 EE16B neither

- Closed book. One 8.5x11 inch page double sided formula sheet. No calculators.
- There are 4 problems worth 100 points total. There may be more time efficient methods to solve problems.

Problem	Points	Score
1	22	
2	25	
3	26	
4	27	
5	27	
TOTAL	100	

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

Tables for reference:

$\tan^{-1}\frac{1}{10} = 5.7°$	$\tan^{-1}\frac{1}{5} = 11.3°$
$\tan^{-1}\frac{1}{4} = 14°$	$\tan^{-1}\frac{1}{3} = 18.4°$
$\tan^{-1}\frac{1}{2} = 26.6°$	$\tan^{-1}\frac{1}{\sqrt{3}} = 30°$
$\tan^{-1} 1 = 45°$	$\tan^{-1}\sqrt{3} = 60°$
$\sin 30° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$
$\cos 45° = \frac{\sqrt{2}}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$

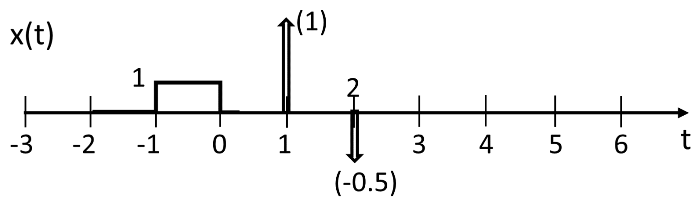
$20\log_{10} 1 = 0dB$	$20\log_{10} 2 = 6dB$	$\pi \approx 3.14$
$20\log_{10}\sqrt{2} = 3dB$	$20\log_{10}\frac{1}{2} = -6dB$	$2\pi \approx 6.28$
$20\log_{10} 5 = 20db - 6dB = 14dB$	$20\log_{10}\sqrt{10} = 10$ dB	$\pi/2 \approx 1.57$
$1/e \approx 0.37$	$\sqrt{10} \approx 3.164$	$\pi/4 \approx 0.79$
$1/e^2 \approx 0.14$	$\sqrt{2} \approx 1.41$	$\sqrt{3} \approx 1.73$
$1/e^3 \approx 0.05$	$1/\sqrt{2} \approx 0.71$	$1/\sqrt{3} \approx 0.58$

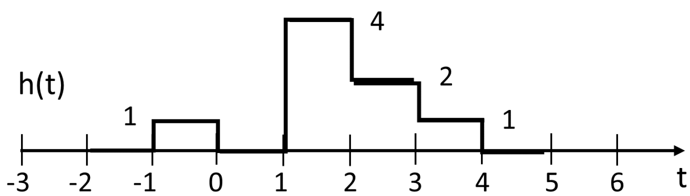
Problem 1 LTI Properties (22 pts)

[16 pts] a. Classify the following systems, with input $x(t)$ or $x[n]$ and output $y(t)$ or $y[n]$. In each column, write "yes", "no", or "?" if the property is not decidable with the given information. (+1 for correct, 0 for blank, -0.5 for incorrect).
Note: $\Pi(t) = u(t + \frac{1}{2}) - u(t - \frac{1}{2})$

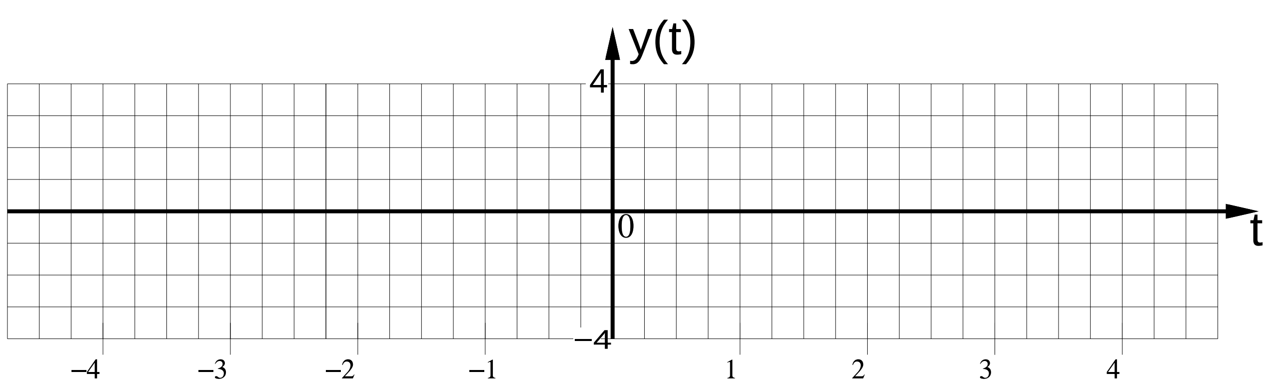
System	Causal	Linear	Time-invariant	BIBO
a. $y(t) = x(t) * \Pi(t)$				
b. $y(t) = x(t) \cdot [\Sigma_{n=-\infty}^{\infty} \delta(t - \frac{n}{2}) * \Pi(t)]$				
c. $y[n] = x[n] \cdot y[n-2] + u[n-2]$				
d. $y(t) = \int_{-1}^{1} x(\tau)\Pi(t-\tau)d\tau$				

[6 pts] e. An LTI system has input $x(t)$ and impulse response $h(t)$ as shown below:



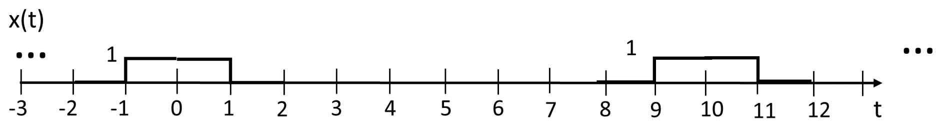


Sketch the output $y(t)$ on the grid below, noting key times and amplitudes.



Problem 2 Fourier Series (25 pts)

You are given a periodic function $x(t)$ as shown, where the shape is a rectangular pulse of height 1 and width 2, centered at $t = 0$:

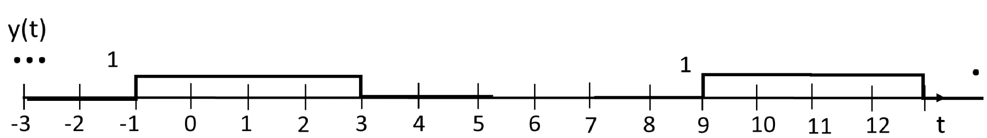


Note that $x(t)$ can be represented by a Fourier Series: $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$.

[1 pts] a. What is the fundamental frequency $\omega_o =$ __________

[8 pts] b. Find $a_k =$ __________________________

Given a new signal $y(t)$ as shown:



Periodic function $y(t)$ can be represented by a Fourier Series: $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_o t}$

[6 pts] d. Find b_k in terms of $a_k =$ __________________________

Problem 2, continued.

[5 pts] e. If $y(t) = x(t) * h(t)$, find $h(t) =$: ______________________

The signal $x(t)$ is passed through an LTI filter $g(t)$ with impulse response:

$$g(t) = \frac{\pi}{3} e^{\frac{-\pi}{3} t} u(t)$$

such that $z(t) = x(t) * g(t)$, where $z(t)$ is also periodic and

$$z(t) = \sum_{k=-\infty}^{\infty} z_k e^{jk\omega_o t},$$

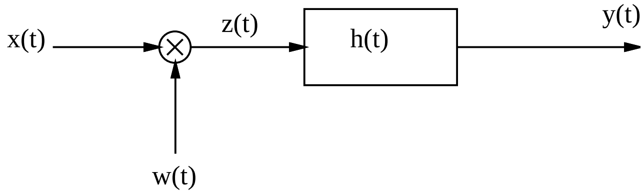
[8 pts] f. Find z_k in terms of a_k = ______________________

[2 pts] g. What is the total time average power in $x(t)$? __________

[5 pts] h. What is the percentage of the total power in $x(t)$ which is not at DC or the fundamental frequency?

percent = __________

Problem 3. Fourier Transform (26 pts)
For each part below, consider the following system:



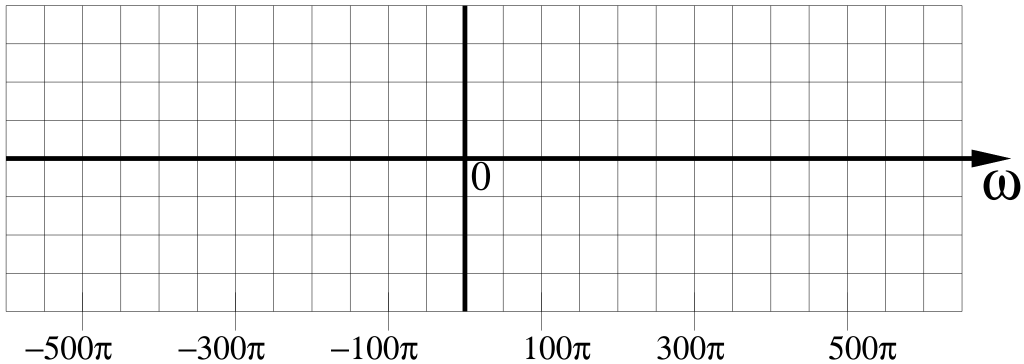
Where $x(t) = \Pi(25t)\cos(300\pi t)$, $\quad w(t) = \cos(250\pi t)$, $\quad h(t) = \frac{2\sin(100\pi t)}{t}$

(Recall that $\Pi(t) = u(t + \frac{1}{2}) - u(t - \frac{1}{2})$.)

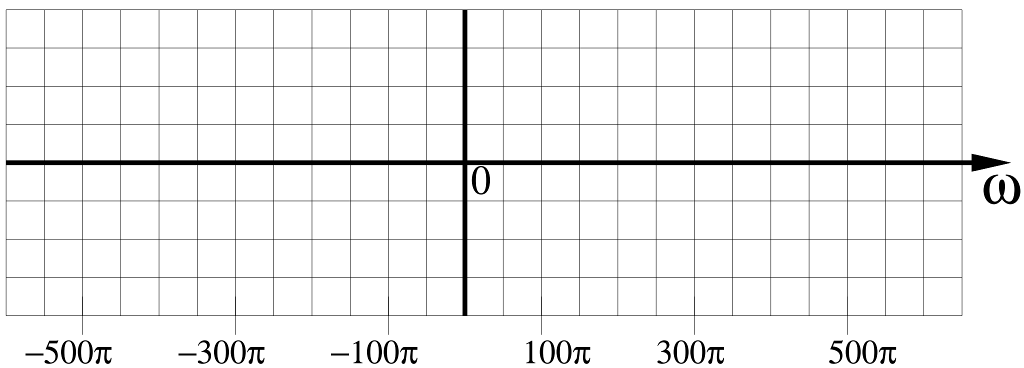
On the next page, sketch $Re\{X(j\omega)\}, Re\{Z(j\omega)\}, Re\{Y(j\omega)\}$ labelling height/area, center frequencies, and key zero crossings for $-500\pi \leq \omega \leq 500\pi$:

Problem 3, continued.

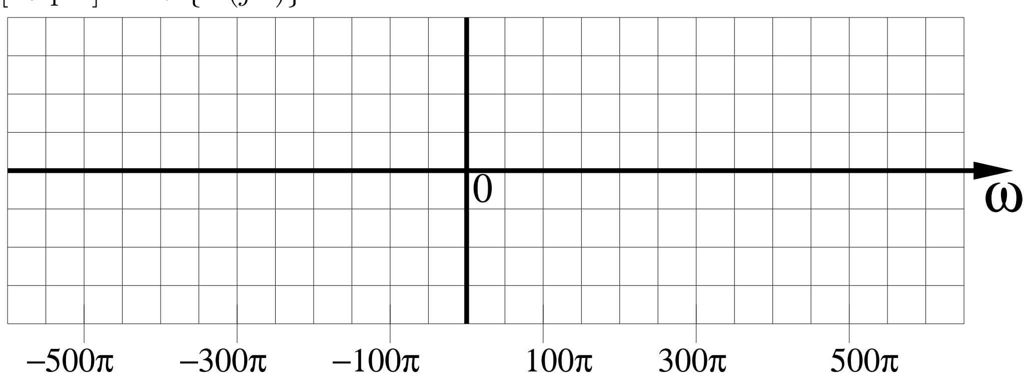
[6 pts] a. $Re\{X(j\omega)\}$



[10 pts] b. $Re\{Z(j\omega)\}$



[10 pts] c. $Re\{Y(j\omega)\}$

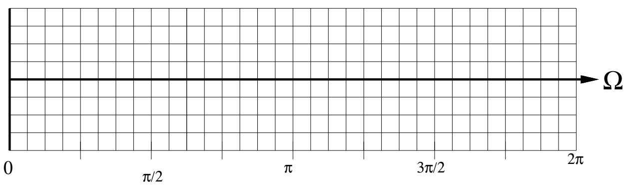


Problem 4. DTFT (27 points)

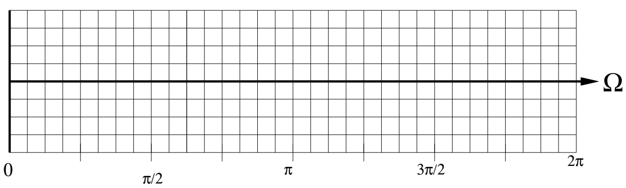
[5 pts] a. Given a discrete time signal $x[n] = \cos(\omega_o n) = \frac{1}{2}\cos(\omega_1 n)$,

find the DTFT $X(e^{j\Omega}) =$_________

[5 pts] b. Sketch $X(e^{j\Omega})$:



[5 pts] c. A causal LTI system with input $x[n]$ has output $y[n]$. Let $y[n]$ have DTFT $Y(e^{j\Omega})$. Then $Y(e^{j\Omega}) = X(e^{j\Omega})H(e^{j\Omega})$. Find and sketch $H(e^{j\Omega})$ such that $y[n] = \cos(\omega_o n)$:



[5 pts] d. Find $h[n]$ for the $H(e^{j\Omega})$ above.

$h[n] =$_________

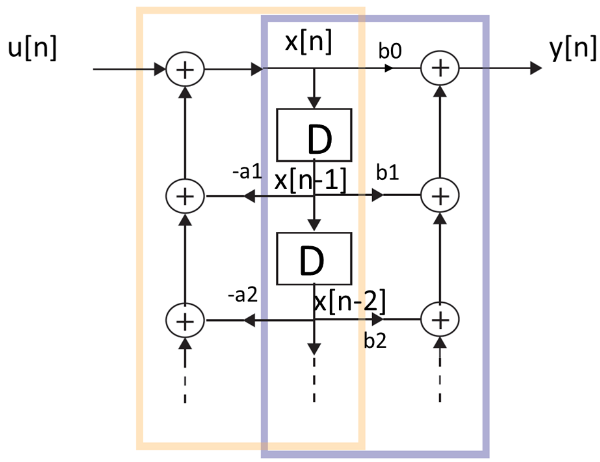
[5 pts] e. Given the difference equation for the LTI causal system with input $u[n]$, and output $y[n]$:

$$y[n] = u[n-2] + \frac{3\sqrt{3}}{4}y[n-1] + \frac{9}{16}y[n-2]$$

For the minimal block diagram below, specify

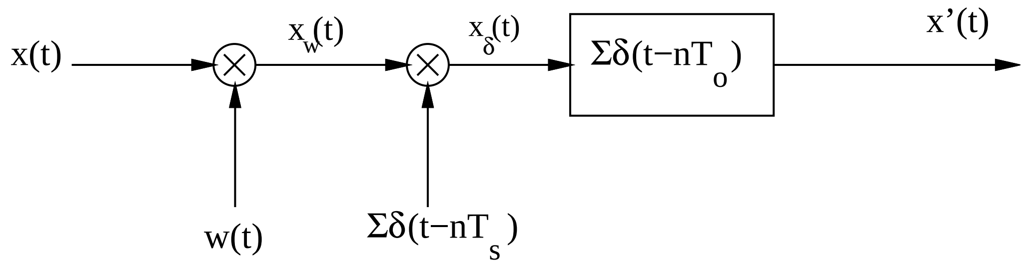
$b_o =$ ___ $b_1 =$ __ $b_2 =$ __

$a_1 =$ __ $a_2 =$ __

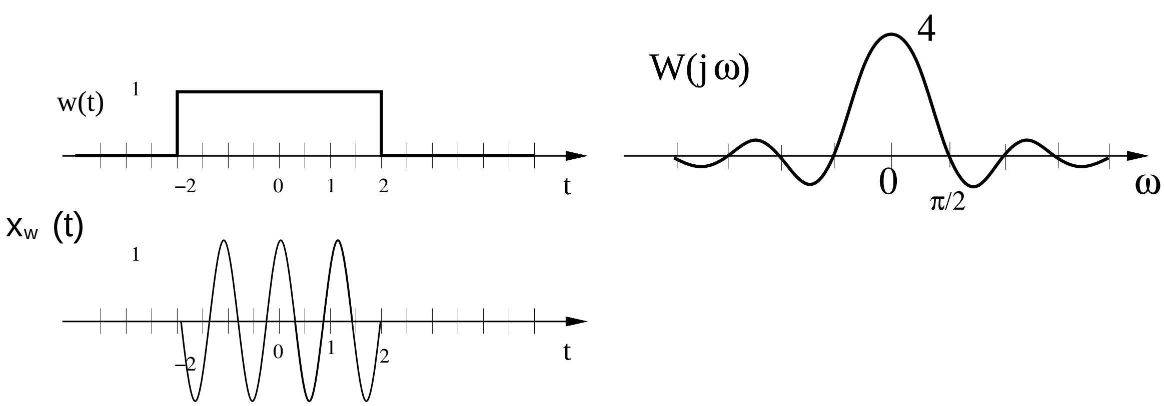

u[n]
x[n]
b0
y[n]
D
-a1
x[n-1]
b1
D
-a2
x[n-2]
b2

Problem 5. Sampling and Discrete Fourier Transform (30 pts)

Consider the system below, where $x(t) = \cos(\frac{3\pi}{2}t)$. Let $T_s = 0.5$ sec, $T_o = 8$ sec, $w(t) = \Pi(t/4)$. Sketches should label peak magnitudes, and frequency of zero crossing(s) should match given scale.
(All time signals are real and even, hence all spectra are also real and even.)
Note $\Pi(t) = u(t + 0.5) - u(t - 0.5)$.
Note that the window has spectrum $W(j\omega) = \frac{2\sin 2\omega}{\omega}$.

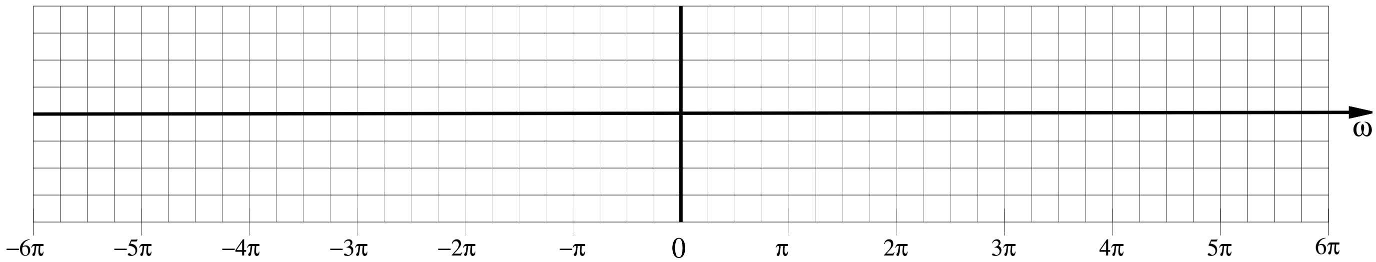


The window function $w(t)$, windowed cosine $x_w(t)$ and $W(j\omega)$ are shown for convenience here:

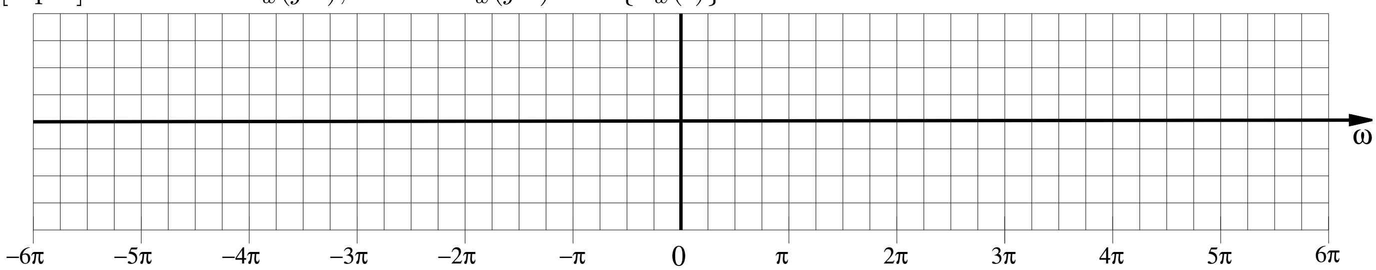


Problem 5. cont.

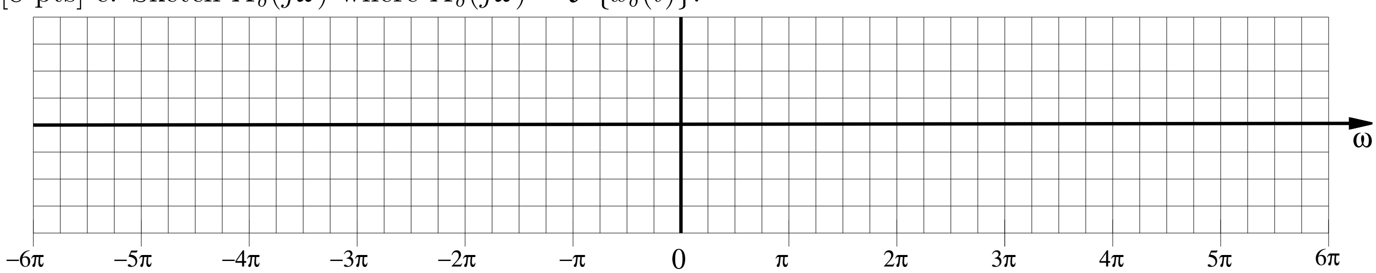
[2 pts] a. Sketch $X(j\omega)$, where $X(j\omega) = \mathcal{F}\{x(t)\}$:



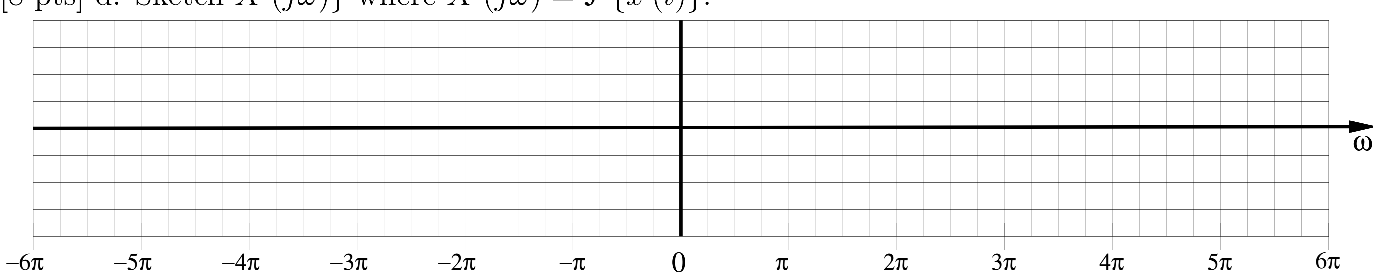
[8 pts] b. Sketch $X_w(j\omega)$, where $X_w(j\omega) = \mathcal{F}\{x_w(t)\}$:



[8 pts] c. Sketch $X_\delta(j\omega)$ where $X_\delta(j\omega) = \mathcal{F}\{x_\delta(t)\}$:

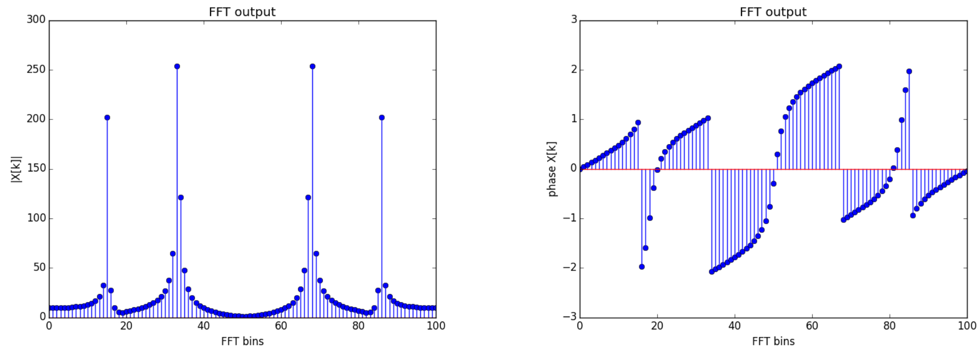


[8 pts] d. Sketch $X'(j\omega)\}$ where $X'(j\omega) = \mathcal{F}\{x'(t)\}$:



Problem 5. cont.

A real bandlimited signal $x(t)$ is sampled with $N = 100$ for 10 seconds, using a rectangular window of width 10 seconds. The DFT of $x[n]$ is calculated using `X = np.fft.fft(x)`. The magnitude and phase of the DFT is shown below.



$|X[k]|$ $\qquad\qquad$ $\angle X[k]$

for samples $X[0]...X[31]$. Using reasoning as in problem 3iv above, explain the differences between the DFT of $x[n]$ and $X(j\omega)$, the FT of $x(t) = \cos(\omega_o t)$. In particular, consider the effects on $X'(j\omega))$ of the window and time shift.

[1 pt] e. What is the spacing of freqency samples $k =$ ___ (rad s^{-1})

Assume $x(t) = a_1 \cos(\omega_1 t + \phi_1) + a_2 \cos(\omega_2 t + \phi_2)$.

[2 pt] f. From the DFT plot, estimate $\omega_1 =$ ___ $\qquad$ $\omega_1 =$ ___

[2 pt] g. From the DFT plot, approximately estimate $a_1 =$ ___ $\qquad$ $a_2 =$ ___