EE 120: Signals and Systems MIDTERM 3
Department of EECS 10 December 2008
UC BERKELEY

LAST Name ______________ FIRST Name ______________

Discussion Time ______________

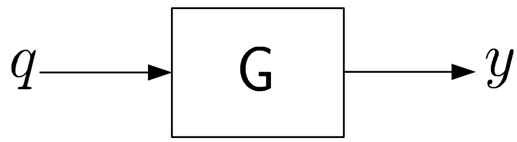
- **(10 Points)** Print your name and discussion time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 90 minutes to complete. You will be given at least 90 minutes, up to a maximum of 110 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except three double-sided $8.5" \times 11"$ sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT3.1 (70 Points) A DC motor is an electromechanical device that converts electrical energy into mechanical energy. A simple model for a DC motor is a *causal* LTI system G described by the transfer function

$$\widehat{G}(s) = \frac{1}{Ms^2 + Ds} = \frac{1/M}{s(s + D/M)}.$$

The parameter M denotes the total moment of inertia and D the total rotational damping.

Throughout this problem, assume $M = 1$ and $D = 16$.

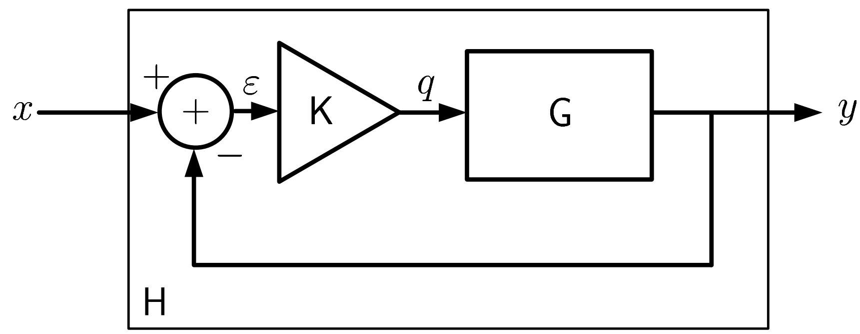


The input to the motor is an electrical signal q, which can be voltage or current, and its corresponding output y is the angular position of the DC motor's rotor.

(a) Is our model of the motor BIBO stable? Explain.

(b) Determine the linear, constant-coefficient differential equation that governs the input-output behavior of the motor G.

We want to use the DC motor to control the angular position of a heavy object. The figure below shows a position control mechanism to do this; it's a feedback configuration with a proportional controller K.



The input signal x represents a *desired* angular position. The output signal y is the *actual* angular position of the object; sensors determine the actual position and feed it back to be subtracted from the desired angular position x.

The *error signal* ε (the difference between the desired and actual positions) is amplified by the proportional controller K, which then sends a command signal q to the motor G.

The transfer function of the proportional controller is simply a positive constant: $\widehat{K}(s) = K > 0$.

(c) Determine a reasonably simple expression for the transfer function of the closed-loop position control system H.

(d) True or False? Explain.

The closed-loop system H is guaranteed to be stable.

(e) We want to change the angular position of the motor (including the object) instantaneously to a desired value, say 1. That is, we apply a unit-step input to the position control system H: $x(t) = u(t)$.

(i) Determine $\widehat{Y}(s)$, the transform of the actual position signal y.

(ii) Let $K = 63$. Determine, and provide a qualitative, and reasonably well-labeled, plot of, the unit-step response $y(t)$, identifying its steady-state and transient components, both in the mathematical expression that you obtain and on the plot.
<u>Hint</u>: Write

$$\widehat{Y}(s) = \frac{A}{s} + \frac{B}{s+\alpha} + \frac{C}{s+\beta}$$

for appropriate constants α and β. Determine A, B, and C. And take the inverse transform to the time domain. And express y in terms of its steady-state and transient components: $y(t) = y_{\mathsf{ss}}(t) + y_{\mathsf{tr}}(t)$.

(iii) Let $K = 128$. Show that the unit-step response y is of the form:

$$y(t) = [\alpha + \beta \cos(\omega_0 t + \phi_0)]u(t).$$

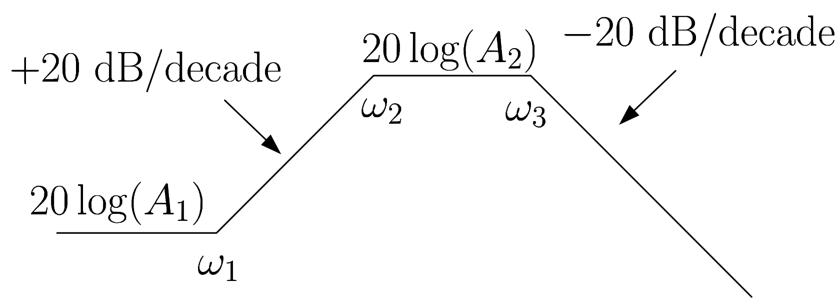
Determine α, β, ω_0, and ϕ_0, and provide a reasonably well-labeled plot of $y(t)$.
Hint: Write

$$\widehat{Y}(s) = \frac{A}{s} + \frac{B}{s+\lambda} + \frac{C}{s+\lambda^*},$$

determine A and B, and explain why $C = B^*$. Then take the inverse transform.

(iv) For what values of K are you guaranteed to not have any ringing effect (damped oscillation or overshoot) in the step-response y?

MT3.2 (35 Points) A Bode plot is shown below.



The frequencies are $\omega_1 = 20\pi$, $\omega_2 = 200\pi$, and $\omega_3 = 2000\pi$. The gains A_1 and A_2 are positive. The following expressions are offered as potential transfer functions for the Bode plot:

$$\widehat{H}_I(s) = \frac{A_1\,(1 + s/\omega_1)}{(1 + s/\omega_2)\,(1 + s/\omega_3)}$$

$$\widehat{H}_{II}(s) = \frac{A_2\,(1 + \omega_1/s)}{(1 + s/\omega_3)\,(1 + \omega_2/s)}$$

$$\widehat{H}_{III}(s) = \frac{A_1\,(1 + \omega_1/s)}{(1 + s/\omega_3)\,(1 + \omega_2/s)}$$

(a) Which candidate transfer function, or functions, correspond correctly with the Bode plot?

(i) Each of $\widehat{H}_I$, $\widehat{H}_{II}$, and $\widehat{H}_{III}$.
(ii) Each of $\widehat{H}_I$ and $\widehat{H}_{II}$.
(iii) Each of $\widehat{H}_I$ and $\widehat{H}_{III}$.
(iv) Each of $\widehat{H}_{II}$ and $\widehat{H}_{III}$.
(v) None of the transfer functions.

(b) Without doing much work, determine $y(0)$.

(c) Determine the numerical value of the ratio A_1/A_2.

You may use the blank space below for scratch work. Nothing written below this line on this page will be considered in evaluating your work.

LAST Name ____________________ FIRST Name ____________________

Discussion Time ____________________

Problem Name	Points 10	Your Score
1	70	
2	35	
Total	**115**	