EECS 120 Signals & Systems — University of California, Berkeley: Fall 2005
Gastpar — November 16, 2005

Solutions to Exam 2

Last name	First name	SID

- You have 1 hour and 45 minutes to complete this exam.
- The exam is closed-book and closed-notes; calculators, computing and communication devices are *not* permitted.
- No form of collaboration between the students is allowed. If you are caught cheating, you may fail the course and face disciplinary consequences.
- However, two handwritten and *not photocopied* double-sided sheet of notes is allowed.
- Additionally, you receive Tables 3.1, 3.2, 4.1, 4.2, 5.1, 5.2, 9.1, 9.2 from the class textbook.
- If we can't read it, we can't grade it.
- We can only give partial credit if you write out your derivations and reasoning in detail.
- You may use the back of the pages of the exam if you need more space.

*** Good Luck! ***

Problem	Points earned	out of
Problem 1		29
Problem 2		28
Problem 3		27
Problem 4		33
Total		117

Problem 1 *(Short Questions.)* 29 Points

(a) (4 Pts) For the system in Figure 1,

$$H(j\omega) = \begin{cases} 1, & \text{for } |\omega| \leq \omega_0 \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Sketch the frequency response $G(j\omega)$ of the overall system between $x(t)$ and $y(t)$.

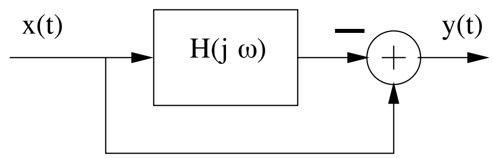
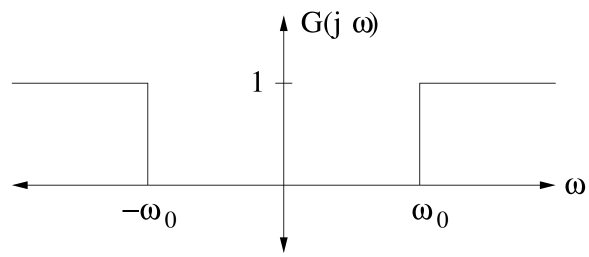


Figure 1:

Solution:

$$\begin{aligned} y(t) &= x(t) - x(t) * h(t) \\ Y(j\omega) &= X(j\omega) - X(j\omega)H(j\omega) = X(j\omega)(1 - H(j\omega)) \\ G(j\omega) &= \frac{Y(j\omega)}{X(j\omega)} = 1 - H(j\omega) \end{aligned}$$

Remark: This problem was a hint for the sampling system design in Problem 3.(c).



(b) (15 Pts) A causal LTI system is described by the following differential equation:

$$\frac{d^2}{dt^2}y(t) + 2\frac{d}{dt}y(t) + 2y(t) = \frac{d^2}{dt^2}x(t) - x(t) \tag{2}$$

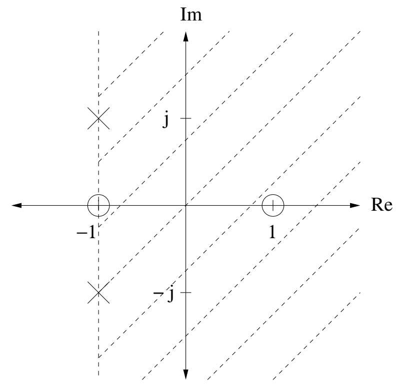
Is this system stable? Does this system have a causal and stable inverse system?

Solution: We take the Laplace transform of both sides of the differential equation to find the transfer function $H(s)$ of the LTI system.

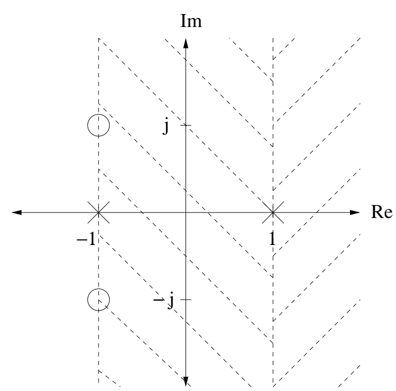
$$s^2Y(s) + 2sY(s) + 2Y(s) = s^2X(s) - X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s^2 - 1}{s^2 + 2s + 2} = \frac{(s+1)(s-1)}{(s - (-1+j))(s - (-1-j))}$$

The system $H(s)$ has poles at $s = -1+j$ and $s = -1-j$, and zeros at $s = 1$ and $s = -1$. Since we are given that $H(s)$ is causal, the region of convergence (ROC) of $H(s)$ is $Re\{s\} > -1$. Thus the ROC of $H(s)$ includes the $j\omega$-axis, which implies that $H(s)$ is stable.



The inverse system $\frac{1}{H(s)}$ has poles at $s = 1$ and $s = -1$, and zeros at $s = -1 + j$ and $s = -1 - j$. The inverse system (which has a rational transfer function) is causal iff the ROC of $\frac{1}{H(s)}$ is the right-half plane. However the inverse system is stable iff the ROC includes the $j\omega$-axis. Therefore the inverse system cannot be both causal and stable.



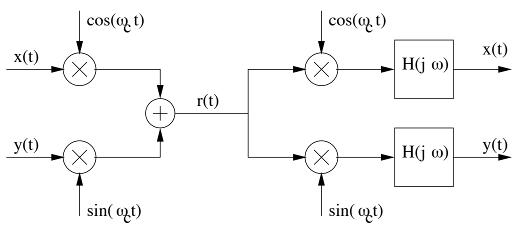


Figure 2: Quadrature modulation.

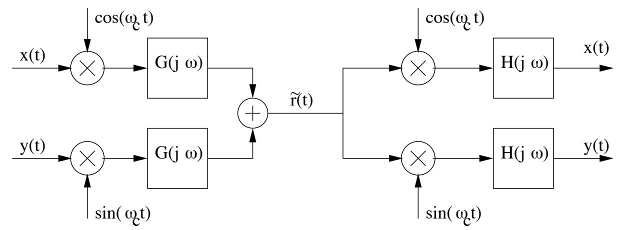


Figure 3: "Improved" quadrature modulation.

(c) (10 Pts) As you have seen in the homework, "quadrature multiplexing" is the system shown in Figure 2, where

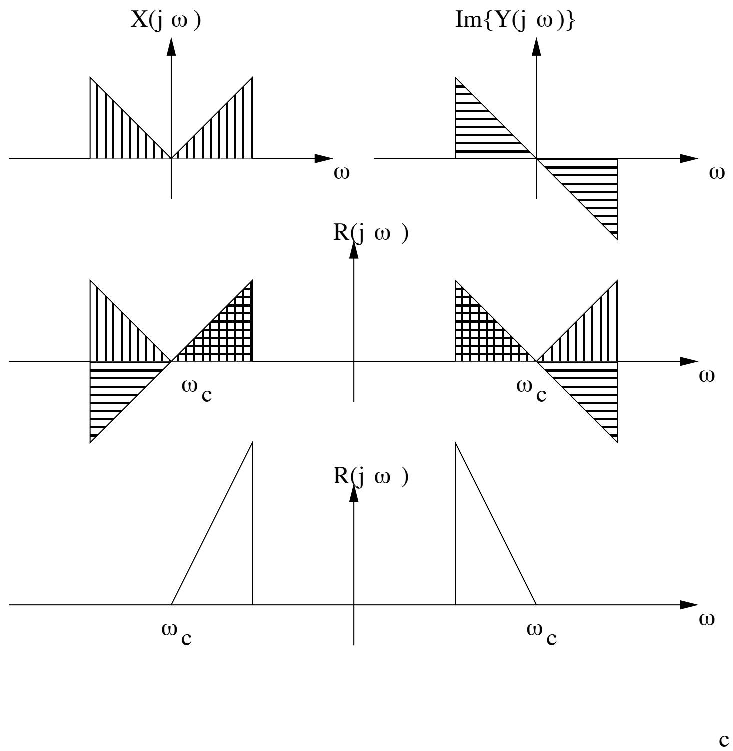
$$H(j\omega) = \begin{cases} 1, & \text{for } |\omega| \leq \omega_M \\ 0, & \text{otherwise.} \end{cases} \quad \text{and} \quad G(j\omega) = \begin{cases} 1, & \text{for } |\omega| \geq \omega_c \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Both original signals are assumed to be bandlimited: $X(j\omega) = Y(j\omega) = 0$, for $|\omega| > \omega_M$; and the carrier frequency is $\omega_c > \omega_M$. The interesting feature is that the effective bandwidth of the signal $r(t)$ is only $2\omega_M$, the same as for a regular AM system with only the signal $x(t)$. Hence, $y(t)$ can ride along for free.

Now, your colleague remembers single-sideband AM and suggests to add the filters $G(j\omega)$ as shown in Figure 3. The effective bandwidth of the transmitted signal $\tilde{r}(t)$ is only ω_M, half as much as in the original quadrature multiplexing system! Show that the "improved" system will not work. *Hint:* Find a pair of example spectra $X(j\omega)$ and $Y(j\omega)$ for which $R(j\omega)$ is *not* zero, but $\tilde{R}(j\omega) = 0$ for all ω. Then, argue (in a few keywords) why this invalidates the "improved" quadrature modulation.

Solution: The basic fact to remember from the homework problem about quadrature modulation is that the spectra overlap and get added up. Consider the example spectra $X(j\omega)$ and $Y(j\omega)$ in the figure below. The spectrum $X(j\omega)$ is purely real-valued. After multiplying by a $\cos(\omega_c t)$, it remains a purely real-valued spectrum. The trick is to select the spectrum $Y(j\omega)$ as purely imaginary; that way, after multiplying by the $\sin(\omega_c t)$, it becomes a purely real-valued spectrum, and hence, there is a chance for it to cancel out the spectrum $X(j\omega)$.

To actually make this happen, we still need to pick the right shape. One example that works is given in the figure below. Note that if $R(j\omega)$ looks as sketched in the figure, then $\tilde{R}(j\omega)$ will be zero — the filter $G(j\omega)$ removes anything below ω_c.



A few remarks: The signal with purely imaginary spectrum as given in $Y(j\omega)$ is a real-valued signal (think of the spectrum of the $\sin$ function - it's purely imaginary). This is because it is conjugate-symmetric ($Y(j\omega) = Y^*(-j\omega)$). Also, there are many other ways to convince your colleague that the system will not work. A longer approach is to select example spectra and to show that demodulation as suggested in Figure 3 will not recover the desired data-carrying signals $x(t)$ and $y(t)$.

Problem 2 *(Discrete-time processing of continuous-time signals.)* 28 Points

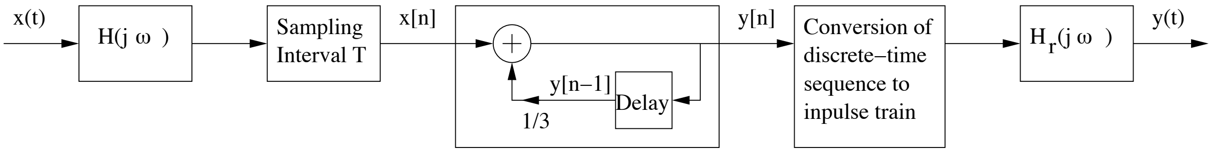


Figure 4:

For the system in Figure 4,

$$H(j\omega) = \begin{cases} 1, & \text{for } |\omega| \le \frac{\pi}{T} \\ 0, & \text{otherwise.} \end{cases} \quad \text{and} \quad H_r(j\omega) = \begin{cases} T, & \text{for } |\omega| \le \frac{\pi}{T} \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

(a) (20 Pts) Give the formula for the overall system response $G(j\omega)$, relating $x(t)$ and $y(t)$. Also give a sketch of the magnitude $|G(j\omega)|$, paying particular attention to the labeling of the frequency axis. *No derivation is necessary to get full credit.*

Solution: The difference equation and frequency response of the discrete-time block can be found as:

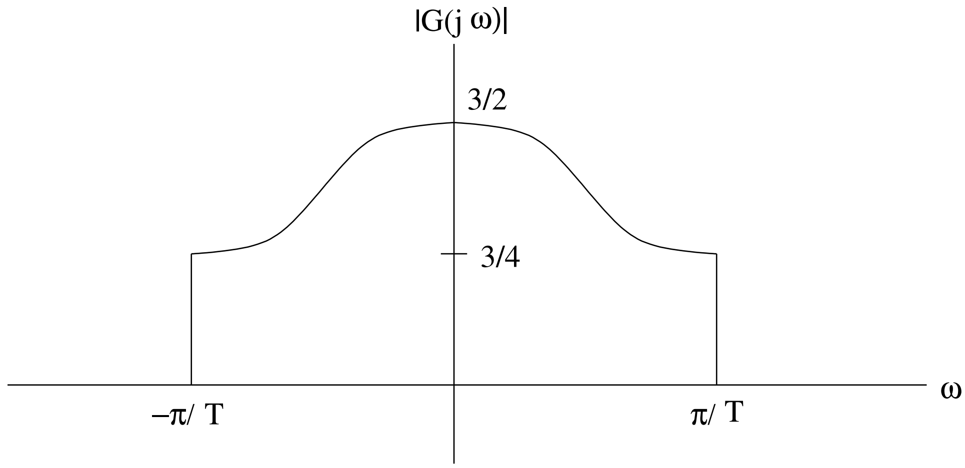
$$\begin{aligned} y[n] &= x[n] + \frac{1}{3} y[n-1] \\ Y\left(e^{j\Omega}\right) &= X\left(e^{j\Omega}\right) + \frac{1}{3} e^{-j\Omega} Y\left(e^{j\Omega}\right) \\ G_d\left(e^{j\Omega}\right) &= \frac{1}{1 - \frac{1}{3} e^{-j\Omega}} \end{aligned}$$

There is no aliasing because of the filter $H(j\omega)$, so the continuous-time system response can be easily found using Equation 7.25 in OWN.

$$G(j\omega) = \begin{cases} \frac{1}{1-\frac{1}{3}e^{-j\omega T}} & |\omega| \le \frac{\pi}{T} \\ 0 & |\omega| > \frac{\pi}{T} \end{cases}$$

The magnitude $|G(j\omega)|$ for $|\omega| < \frac{\pi}{T}$ can be found as follows:

$$
\begin{aligned}
|G(j\omega)| &= \frac{1}{|1-\frac{1}{3}e^{-j\omega T}|} \\
&= \frac{1}{|1-\frac{1}{3}\left(\cos(-\omega T)+j\sin(-\omega T)\right)|} \\
&= \frac{1}{|\left(1-\frac{1}{3}\cos(\omega T)\right)+j\left(\frac{1}{3}\sin(\omega T)\right)|} \\
&= \frac{1}{\sqrt{1-\frac{2}{3}\cos(\omega T)+\frac{1}{9}\cos^2(\omega T)+\frac{1}{9}\sin^2(\omega T)}} \\
&= \frac{1}{\sqrt{\frac{10}{9}-\frac{2}{3}\cos(\omega T)}}
\end{aligned}
$$



(b) (8 Pts) For $x(t) = e^{j\pi t/(2T)}$, determine the corresponding output signal $y(t)$. Your answer should not contain an integral, but apart from that, there is no need to simplify it down.

Solution: After writing $x(t) = e^{j(\pi/(2T))t}$ we see that:

$$
y(t) = G\left(j\frac{\pi}{2T}\right)e^{j(\pi/(2T))t} = \frac{1}{1-\frac{1}{3}e^{-j\pi/2}}e^{j\pi t/(2T)} = \frac{1}{1+j\frac{1}{3}}e^{j\pi t/(2T)}
$$

Problem 3 *(Sampling System Design.)* 27 Points

The signal $x(t)$ has the Fourier transform shown in Figure 5.

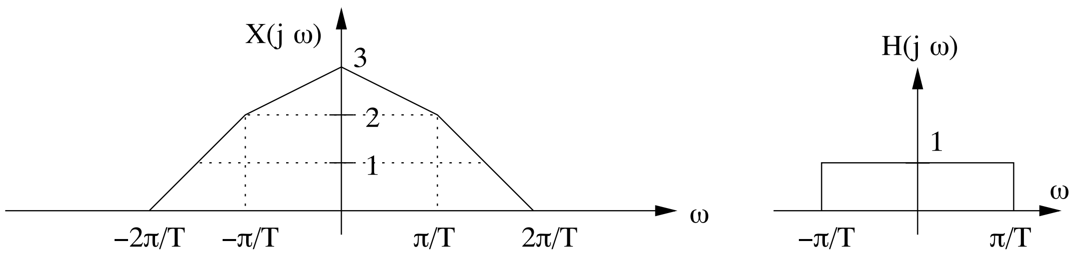
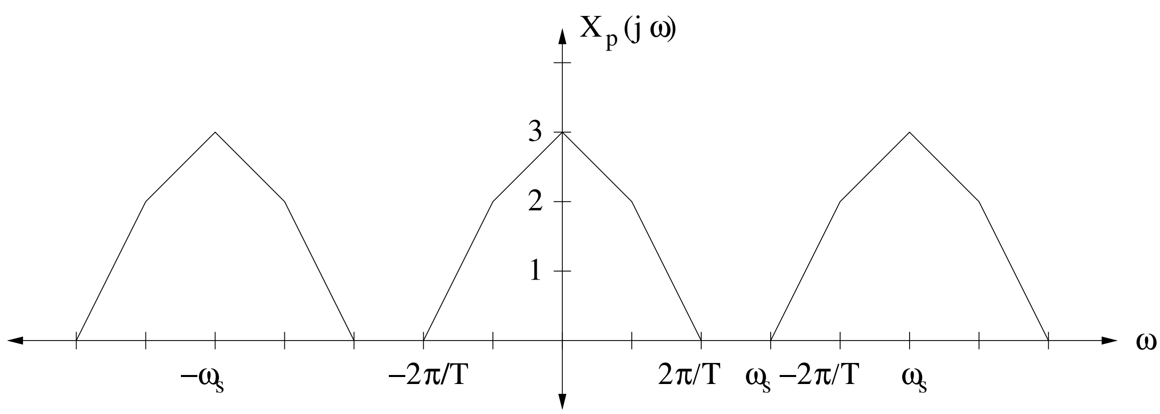


Figure 5:

(a) (5 Pts) As a function of T (as in Figure 5), determine the smallest sampling frequency $\omega_s = 2\pi/T_s$ (where T_s is the sampling interval) for which perfect reconstruction can be guaranteed for the signal $x(t)$. A graphical justification (sketch with labels on the frequency axis) is sufficient.

Solution: By the sampling theorem, the bandlimited signal $x(t)$, with $X(j\omega) = 0$ for $|\omega| > \frac{2\pi}{T}$, can be perfectly reconstructed if we sample at frequency $\omega_s \geq 2\left(\frac{2\pi}{T}\right) = \frac{4\pi}{T}$. Graphically, we see that the shifted replica of $X(j\omega)$ that is centered at ω_s does not overlap with the replica at 0 if $\omega_s - \frac{2\pi}{T} \geq \frac{2\pi}{T}$.



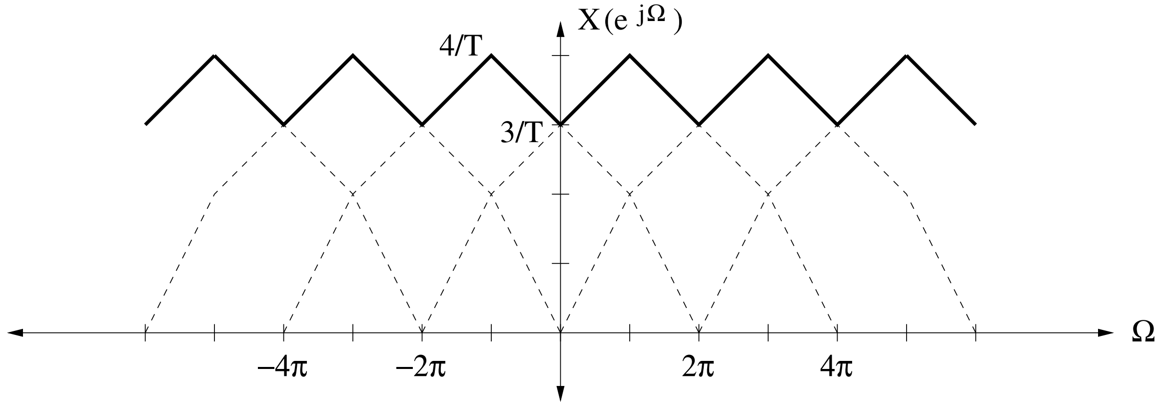
(b) (10 Pts) Consider the signal $y(t) = h(t) * x(t)$, where $h(t)$ is the impulse response of the filter $H(j\omega)$ in Figure 5. Sketch the spectra of the two discrete-time signals

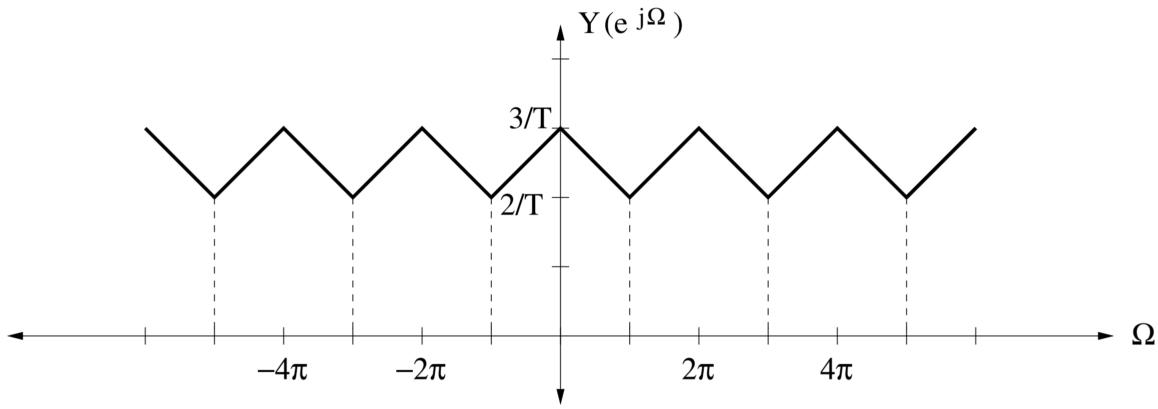
$$x[n] = x(nT) \quad (5)$$

$$y[n] = y(nT), \quad (6)$$

where T is the *same* as in Figure 5. Which effect explains the difference between $x[n]$ and $y[n]$?

Solution: Now we sample the signals $x(t)$ and $y(t)$ with frequency $\frac{2\pi}{T}$, and then convert the resulting impulse trains $x_p(t)$ and $y_p(t)$ to discrete-time signals $x[n]$ and $y[n]$. In the frequency domain, $X_p(j\omega) = \frac{1}{T}\sum_k X(j(\omega - 2\pi k/T))$ and $X(e^{j\Omega}) = X_p(j\frac{\Omega}{T})$ (and similarly for $Y(e^{j\Omega})$). Note that sampling scales the vertical axis by $\frac{1}{T}$, and converting to discrete-time scales the frequency axis by T.





Since we are sampling $x(t)$ at a frequency less than $\omega_s = \frac{4\pi}{T}$, the shifted replicas of $X(j\omega)$ overlap, producing the aliasing effect in $X(e^{j\Omega})$. In contrast, because $y(t)$ is bandlimited by $\frac{\pi}{T}$, the shifted replicas of $Y(j\omega)$ do not overlap, and there is no aliasing effect in the spectrum of $Y(e^{j\Omega})$.

(c) (12 Pts) The goal is now to implement a sampler with sampling interval $T_0 = T/2$, where T is as in Figure 5. Unfortunately, such a fast sampler is not available in the current technology. Instead, you have access to the following devices:

- samplers with sampling interval T, where T is the *same* as in Figure 5 (any number)
- anti-aliasing filters with the frequency response given in Figure 5 (any number)
- continuous-time signal adders/subtractors (any number)
- *any* discrete-time processing devices (ideal filters included).

Draw the block diagram of a system that takes as an input the signal $x(t)$ (with spectrum as shown in Figure 5) as outputs the discrete-time signal $x_0[n] = x(nT_0)$. *Hint:* To maximize your chance of partial credit, give spectral plots of intermediate signals in your system.

Solution: The key insight is that we have to sample different parts of the signal separately. Clearly, we can low-pass filter the signal $x(t)$ to obtain the signal $y(t)$ in the figure below. Since $y(t)$ is now bandlimited to π/T, we can sample it with our sampler (interval T) with no aliasing. Separately, we want to sample the "high-pass" part of the signal $x(t)$. Here, we can use two standard tricks: first, to get the high-pass part, we can just take $x(t)$ and *subtract out* the low-pass part, leading to the signal $z(t)$ in the figure below. Second, in this case, the resulting signal can be sampled directly with sampling interval T, with no aliasing. This is just like in the homework problem about band-pass sampling. Alternatively, if we had access to a continuous-time mixer (multiplication by a cosine function), we could modulate it down before sampling (but as we said, for our example case, this is not necessary).

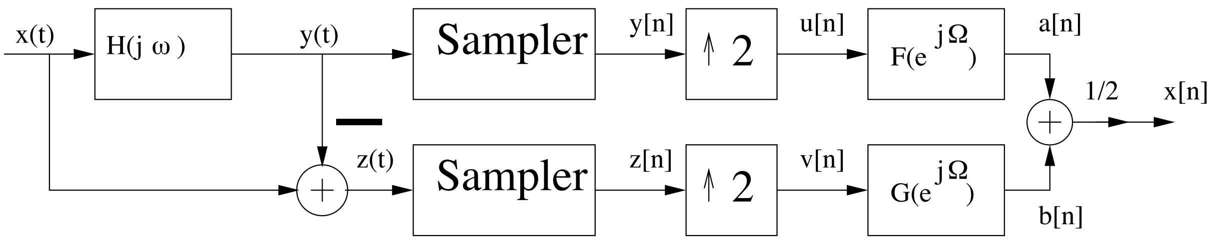
The resulting signals $y[n]$ and $z[n]$ now have to be combined. The main insight at this point is that we need twice the sampling rate in the end, and so, we should upsample both $y[n]$ and $z[n]$.

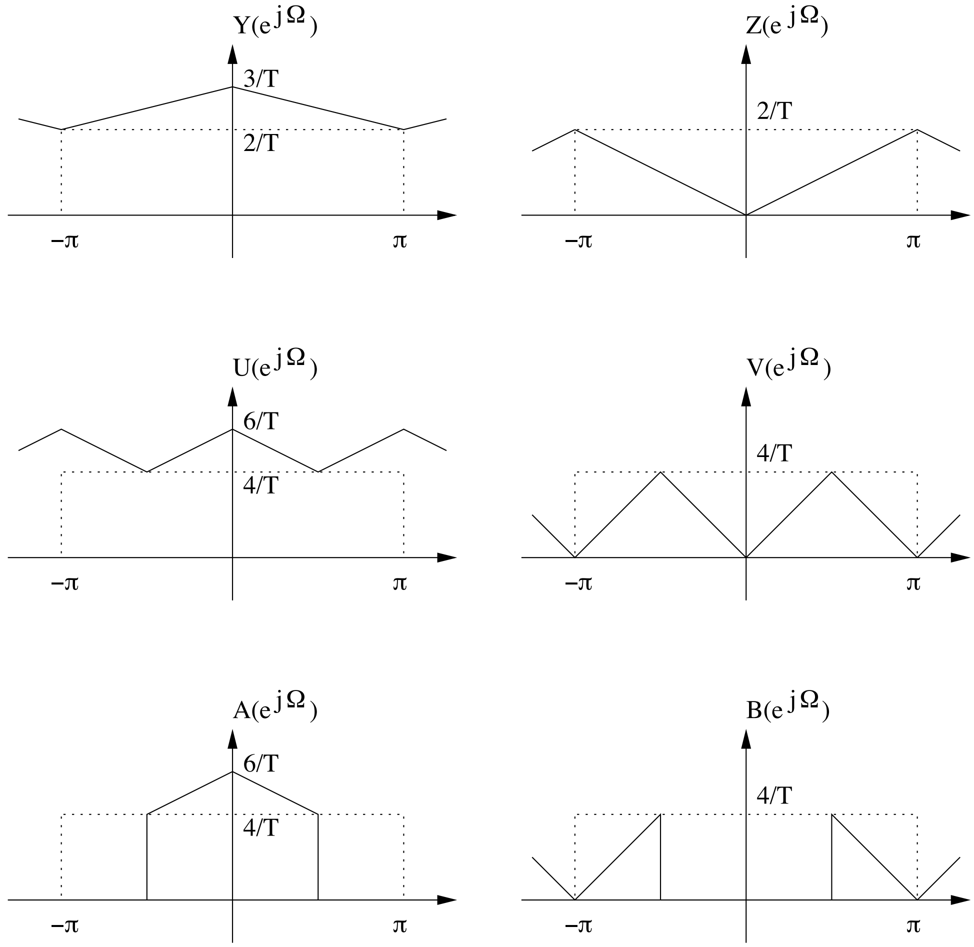
To see how to implement the combining, one really needs the spectral plots of the upsampled signals, $U(e^{j\Omega})$ and $V(e^{j\Omega})$: It is immediately clear that we want to low-pass filter $U(e^{j\Omega})$ and high-pass filter $V(e^{j\Omega})$. Let's define:

$$F(e^{j\Omega}) = \begin{cases} 1, & \text{for } |\Omega| \leq \frac{\pi}{2} \\ 0, & \text{otherwise.} \end{cases} \quad \text{and} \quad G(e^{j\Omega}) = \begin{cases} T, & \text{for } |\Omega| > \frac{\pi}{2} \\ 0, & \text{otherwise.} \end{cases}$$

Recall that these are really 2π-periodic; we are merely specifying one period. The spectral plots in the figure on the next page shows that the system below works.

Grading: Most of the points were given for the two key ideas, namely, that we need two different signal paths, and that we need upsampling.




Y(e^{jΩ})
3/T
2/T
−π
π
Z(e^{jΩ})
2/T
−π
π
U(e^{jΩ})
6/T
4/T
−π
π
V(e^{jΩ})
4/T
−π
π
A(e^{jΩ})
6/T
4/T
−π
π
B(e^{jΩ})
4/T
−π
π

Problem 4 *(PAM.)* 33 Points

Two pulses are suggested for a PAM system:

$$q_1(t) = \begin{cases} 1, & |t| \le T/4 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad q_2(t) = \begin{cases} -1 & -T/4 \le t < 0 \\ 1, & 0 \le t \le T/4 \\ 0, & \text{otherwise} \end{cases}$$

The PAM signal is then

$$x_m(t) = \sum_{n=-\infty}^{\infty} s[n] q_m(t - nT), \text{ for } m = 1, 2. \tag{7}$$

Throughout this problem, **we assume that the data signal is merely $s[n] = 1$, for all n.**

(a) (6 Pts) Find the powers P_1 and P_2 of the two PAM signals $x_1(t)$ and $x_2(t)$. **Solution:** Because $x_1(t)$ and $x_2(t)$ are periodic, we can calculate their power by looking at one period.

$$P_1 = \frac{1}{T}\int_{-T/2}^{T/2} |x_1(t)|^2 dt = \frac{1}{T}\int_{-T/4}^{T/4} 1 dt = \frac{1}{2}$$

$$P_2 = \frac{1}{T}\int_{-T/2}^{T/2} |x_2(t)|^2 dt = \frac{1}{T}\int_{-T/4}^{T/4} 1 dt = \frac{1}{2}$$

(b) (10 Pts) Give the formula for the Fourier series coefficients of the signal $x_1(t)$, and explicitly evaluate the coefficients a_0, a_1 and a_{-1}. Then, do the same for the signal $x_2(t)$.

Solution: The coefficients of $x_1(t)$ are in Table 4.2, 6th entry. For us, $T_1 = T/4$ and $\omega_0 = 2\pi/T$, hence

$$a_k = \frac{\sin(k\pi/2)}{k\pi} \tag{8}$$

where the coefficient at $k = 0$ can be found as usual as the limit of the above expression as $k \to 0$, or by noting that the above formula can be written as $a_k = \frac{1}{2}\text{sinc}(k/2)$, where we remember that $\text{sinc}(0) = 1$. Thus, $a_0 = 1/2$, $a_1 = \frac{1}{\pi}$ and $a_{-1} = \frac{1}{\pi}$.

Alternatively, you can evaluate by hand:

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x_1(t) dt = \frac{1}{2}$$

$$\begin{aligned} a_k &= \frac{1}{T}\int_{-T/2}^{T/2} x_1(t) e^{-jk(2\pi/T)t} dt \\ &= \frac{1}{T}\int_{-T/4}^{T/4} e^{-jk(2\pi/T)t} dt \\ &= \frac{1}{T}\frac{1}{-jk2\pi/T}\left(e^{-jk\pi/2} - e^{jk\pi/2}\right) \\ &= \frac{1}{k\pi}\sin(k\pi/2) \end{aligned}$$

For the signal $x_2(t)$, let's first consider a symmetric box of width $T/4$, centered at zero, and repeated at intervals of T. Call this signal $v(t)$. Hence, again from the table, with $T_1 = 1/8$, we find the FS coefficients of $v(t)$:

$$c_k = \frac{\sin(k\pi/4)}{k\pi} = \frac{1}{4}\text{sinc}(k/4). \tag{9}$$

Clearly, $x_2(t) = v(t - T/8) - v(t + T/8)$, and hence, using the second property (time shifting) in Table 3.1, we find the FS coefficients of the signal $x_2(t)$ as

$$\begin{aligned} b_k &= c_k e^{-jk\pi/4} - c_k e^{jk\pi/4} &(10)\\ &= \frac{1}{4}\text{sinc}(k/4)(-2j)\sin(k\pi/4) &(11)\\ &= -\frac{j}{2}\text{sinc}(k/4)\sin(k\pi/4) &(12)\end{aligned}$$

Thus, $b_0 = 0$, $b_1 = -\frac{j}{\pi}$ and $b_{-1} = \frac{j}{\pi}$.

Alternatively, you can evaluate by hand:

$$b_0 = \frac{1}{T}\int_{-T/2}^{T/2} x_2(t)dt = 0$$

$$\begin{aligned} b_k &= \frac{1}{T}\int_{-T/2}^{T/2} x_2(t)e^{-jk(2\pi/T)t}dt \\ &= \frac{1}{T}\int_{-T/4}^{0} -e^{-jk(2\pi/T)t}dt + \frac{1}{T}\int_{0}^{T/4} e^{-jk(2\pi/T)t}dt \\ &= \frac{1}{T}\frac{1}{jk(2\pi/T)}\left(1 - e^{jk\pi/2}\right) - \frac{1}{T}\frac{1}{jk(2\pi/T)}\left(e^{-jk\pi/2} - 1\right) \\ &= \frac{1}{jk2\pi}\left(2 - e^{jk\pi/2} - e^{-jk\pi/2}\right) \\ &= \frac{1}{jk\pi}\left(1 - \cos(k\pi/2)\right) \end{aligned}$$

Exercise: Show that the above two formulas for b_k are, in fact, equal.

(c) (7 Pts) To actually transmit our PAM signal, we first low-pass filter it:

$$\tilde{x}_m(t) \;=\; h(t) * x_m(t), \text{ where } H(j\omega) = \begin{cases} 1, & \text{for } |\omega| \le \frac{10\pi}{T} \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

Then, we transmit the signals $y_1(t) = \tilde{x}_1(t)\cos(\frac{40\pi}{T}t)$ and $y_2(t) = \tilde{x}_2(t)\cos(\frac{40\pi}{T}t)$. Sketch the Fourier transforms of these two signals in the plots provided below, carefully labeling the frequency axis. In the magnitude plots (i.e., $|Y_1(j\omega)|$ and $|Y_2(j\omega)|$, respectively), the amplitudes need not be exact. *Remark:* The current labels on the frequency axis in the plots are for your convenience only. If you prefer, you can cross them out and start from scratch.

Solution: Since $x_1(t)$ is periodic, its spectrum $X_1(j\omega)$ is a "line spectrum", i.e., composed of delta functions. From Table 4.2, first entry, the delta functions are spaced $2\pi/T$ apart (the fundamental frequency of $x_1(t)$), and their amplitudes are $2\pi a_k$, where a_k are the Fourier series coefficients of $x_1(t)$:

$$X_1(j\omega) \;=\; 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\frac{2\pi}{T}). \tag{14}$$

The low-pass filtering throws away anything at frequencies higher than $10\pi/T$, i.e., only the center and 5 delta functions on each side of the origin survive, but otherwise, the spectrum $\tilde{X}_1(j\omega)$ looks exactly like the spectrum of $X_1(\omega)$:

$$\tilde{X}_1(j\omega) \;=\; 2\pi \sum_{k=-5}^{5} a_k \delta(\omega - k\frac{2\pi}{T}). \tag{15}$$

Multiplying by $\cos(\frac{40\pi}{T}t)$ simply places two copies of the spectrum of $\tilde{X}_1(j\omega)$, one centered at $40\pi/T$, the other at $-40\pi/T$. To get the amplitudes right (but no points were taken off for minor errors in the amplitudes), remember that the spectrum of the cosine consists of two delta functions of amplitude π each, and the multiplication property has a factor of $1/(2\pi)$, hence,

$$Y_1(j\omega) \;=\; \frac{2\pi\cdot\pi}{2\pi}\left(\sum_{k=-5}^{5} a_k\delta(\omega - k\frac{2\pi}{T} - \frac{40\pi}{T}) + \sum_{k=-5}^{5} a_k\delta(\omega - k\frac{2\pi}{T} + \frac{40\pi}{T})\right). \tag{16}$$

For the signal $y_2(t)$, the argument is exactly the same, leading to

$$Y_2(j\omega) \;=\; \frac{2\pi\cdot\pi}{2\pi}\left(\sum_{k=-5}^{5} b_k\delta(\omega - k\frac{2\pi}{T} - \frac{40\pi}{T}) + \sum_{k=-5}^{5} b_k\delta(\omega - k\frac{2\pi}{T} + \frac{40\pi}{T})\right). \tag{17}$$

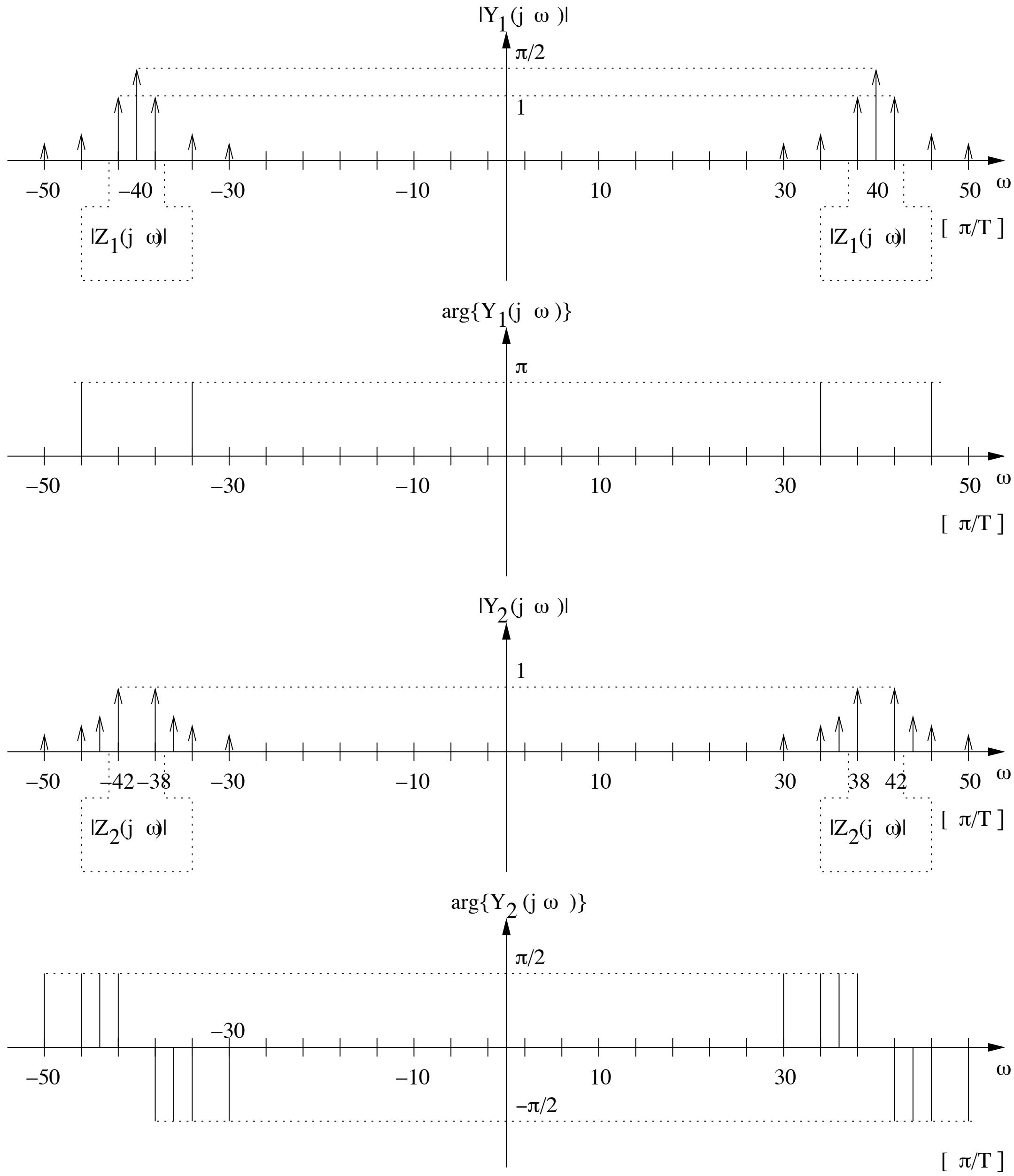

|Y1(j ω)|
π/2
1
−50
−40
−30
−10
10
30
40
50
ω
[π/T]
|Z1(j ω)|
|Z1(j ω)|
arg{Y1(j ω)}
π
−50
−30
−10
10
30
50
ω
[π/T]
|Y2(j ω)|
1
−50
−42
−38
−30
−10
10
30
38
42
50
ω
[π/T]
|Z2(j ω)|
|Z2(j ω)|
arg{Y2(j ω)}
π/2
−30
−50
−10
10
30
ω
−π/2
[π/T]

Figure 6:

(d) (10 Pts) The communication channel's effect on the signal can be described by the following band-pass filter:

$$H_{channel}(j\omega) = \begin{cases} \sin^2(\omega T/4 - \pi) & \text{for } 36\pi/T < |\omega| < 38\pi/T \\ 1, & \text{for } 38\pi/T \le |\omega| \le 42\pi/T \\ \sin^2(\omega T/4 - \pi) & \text{for } 42\pi/T < |\omega| < 44\pi/T \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

The channel output signal is then $z_1(t) = y_1(t) * h_{channel}(t)$ and $z_2(t) = y_2(t) * h_{channel}(t)$, respectively. Assuming that $s[n] = 1$, for all n, find the power of $z_1(t)$ and $z_2(t)$. These are the received powers. Which pulse is more efficient for transmission across this channel?

Solution: The key insight is that $z_1(t)$ is still a periodic signal. Hence, its power is calculated as

$$P_{r1} = \frac{1}{T}\int_T |z_1(t)|^2 dt. \quad (19)$$

From Parseval, this is the same as

$$P_{r1} = \sum_{n=-\infty}^{\infty} |c_k|^2, \quad (20)$$

where c_k are the Fourier series coefficients of the signal $z_1(t)$. From Part *(c)*, we know the Fourier Transform $Z_1(j\omega)$: Passing $Y_1(j\omega)$ through the bandpass filter, only 6 delta pulses survive, namely the three centered around $40\pi/T$ and the three centered around $-40\pi/T$. To find the Fourier series coefficients, we have to divide the amplitude of the delta function by 2π (see Table 4.2, first entry), and so we can read directly our of the figure:

$$|c_{20}| = |c_{-20}| = 1/4, \quad |c_{19}| = |c_{-19}| = c_{21} = c_{-21} = 1/(2\pi), \quad (21)$$

and *all other Fourier series coefficients are zero*, from which we find

$$P_{r1} = \sum_{n=-\infty}^{\infty} |c_k|^2 = 2 \cdot 1/16 + 4 \cdot 1/(2\pi)^2 = \frac{1}{8} + \frac{1}{\pi^2}. \quad (22)$$

By the same token, we can read out the Fourier series coefficients for the signal $z_2(t)$, and find

$$P_{r2} = \sum_{n=-\infty}^{\infty} |d_k|^2 = 4 \cdot 1/(2\pi)^2 = \frac{1}{\pi^2}. \quad (23)$$

Hence, the received power for the pulse $x_1(t)$ is higher, and we conclude that this pulse is more efficient.

Note: There is also a subtlety that we swept under the carpet. Really, it is not clear whether both pulses have the same *transmitted* power — We only calculated the powers of $x_1(t)$ and $x_2(t)$, respectively, but the transmitted signals, really, are $y_1(t)$ and $y_2(t)$. These powers are most easily found along the lines of the above calculation, but we would need some more of the Fourier series coefficients.