University of California, Berkeley

EE118/EE218A Introduction to Optical Engineering **Fall 2014**

Final Exam (5 questions, 50 points, 8 pages)

1. **(10 points) Short answer questions.**

   (a) True/False: The wave equation only applies to propagation of sinusoidal waves.

   (b) True/False: Refractive index can vary with wavelength of light.

   (c) If I want to take a photo during the night, which $f/\#$ should I use to make the image look brighter? f/2.5 or f/5? Why?

   (d) True/False: Nearsighted people need coverging lenses to form a focused image.

   (e) True/False: The speed of light is slower inside water because wavelength is shorter in water.

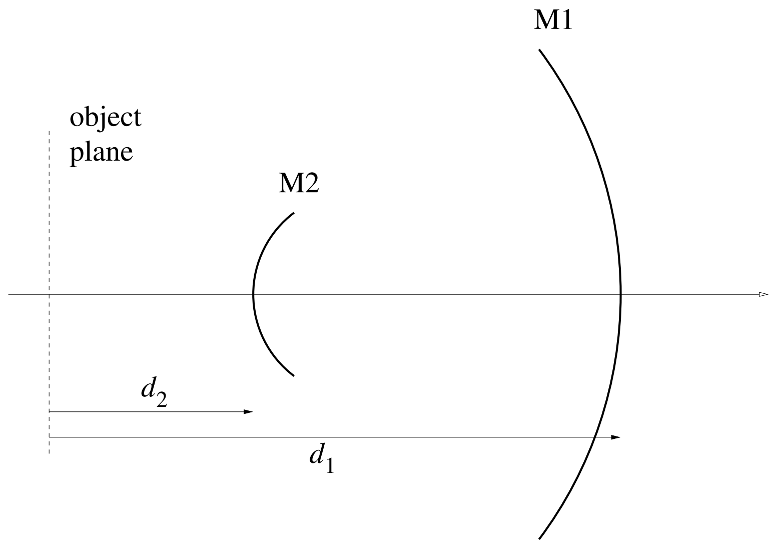
   (f) Can I add two elliptically polarized waves and get a linearly polarized wave?

   (g) The electric field of unpolarized light: a) has no direction at any time, b) rotates rapidly, c) is always parallel to the direction of propagation, d) changes direction randomly and often, e) remains along the same line but reverses direction randomly and often.

   (h) Polarization experiments provide evidence that light is: a) a longitudinal wave, b) a stream of particles, c) a transverse wave, d) some type of wave, e) nearly monochromatic.

   (i) The units of index of refraction are: a) m/s, b) s/m, c) radians, d) m/s^2, e) none of these

2. **(10 points) Two concave mirrors.** Consider an optical system with two concave (converging) mirrors, M1 and M2 as shown in the figure below. M1 is placed at a distance d_1 to the right of an object with its reflective surface facing the object. M2 is placed at a distance d_2 to the right of the object with its reflective surface facing away from the object. M1 has focal length f_1 and M2 has focal length f_2. Assume that M2 has much smaller aperture than M1 and state clearly any further assumptions needed for your analysis.



(a) Ray trace for a system where: $d_1 = 45\text{cm}$, $d_2 = 25\text{cm}$, $f_1 = 20\text{cm}$, $f_2 = 5\text{cm}$. Make sure to indicate the location of the final image and any intermediate images if they occur in this system.

(b) For this same system, write the ABCD matrix representing the entire optical system, from object to image plane?

(c) For this same system, what is the value of the location where the final image forms?

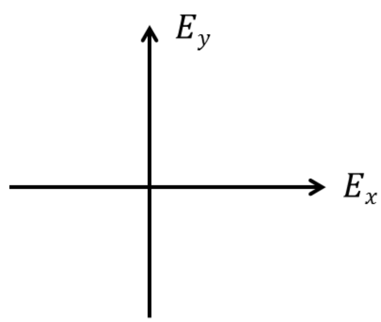
(d) Is the image formed real or virtual? Upright or inverted?

3. **(10 points) Polarizing and Polarizing Prisms.**

(a) Which of the following waves represent (circle one) right-hand circularly polarized light? Why?

(i) $\vec{E} = \hat{x}E_0 cos(kz - \omega t) + \hat{y}E_0 cos(kz - \omega t + \pi)$
(ii) $\vec{E} = \hat{x}E_0 cos(kz - \omega t) + \hat{y}E_0 cos(kz - \omega t + \frac{3\pi}{8})$
(iii) $\vec{E} = \hat{x}E_0 cos(kz - \omega t) + \hat{y}E_0 cos(kz - \omega t - \frac{\pi}{2})$
(iv) $\vec{E} = \hat{x}E_0 cos(kz - \omega t) + \hat{y}E_0 cos(kz - \omega t + \frac{\pi}{2})$

(b) We want to turn the right-hand circularly polarized wave into a linearly polarized wave having an angle 45 degrees relative to the $-\hat{x}$ axis on the xy-plane. What is the phase shift you have to add to the y component of the electric field? Write down the expression of this new field and draw its orientation in the E_x, E_y plot below.



(c) If we want to separate these two components in order to create a polarizing beam splitter, we can place two prisms of birefringent material in the orientation shown in Fig. 1. Given the directions of the optical axis (OA) of the material, which of the crystals (a,b or c) in Fig. 1 can have the largest separation of polarizations after the light passes through the center of it? Explain why. For all crystals, the refractive index along the optical axis (OA) is 1.45, and the refractive index perpendicular to the optical axis is 1.65.

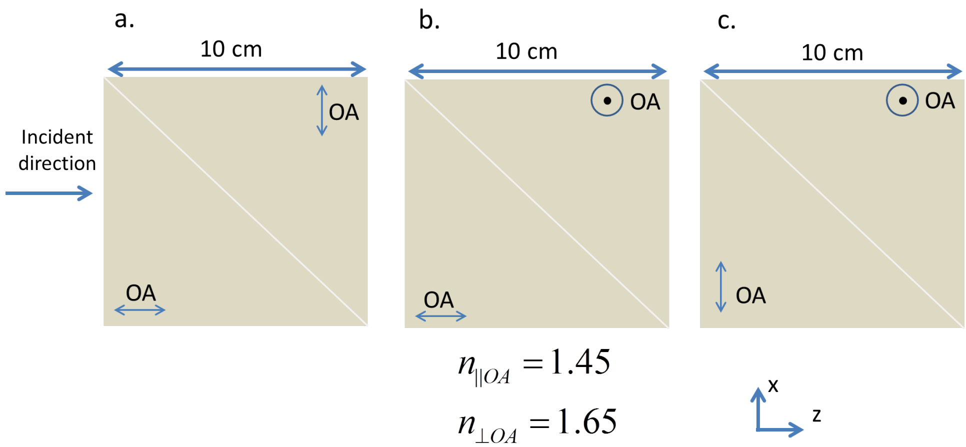


Figure 1: Schematic diagram of three different polarizing prisms.

(d) What is the distance between the polarizations at the edge of the crystal? Show the position of each component after they pass through the crystal that you choose in part (c).

4. **(8 points) Projection Lithogrpahy.** For the semiconductor industry, lithography is one of the key processes to build transistors. A pattern on the mask (object plane)is imaged to the wafer (image plane) for printing (see Fig. 2).

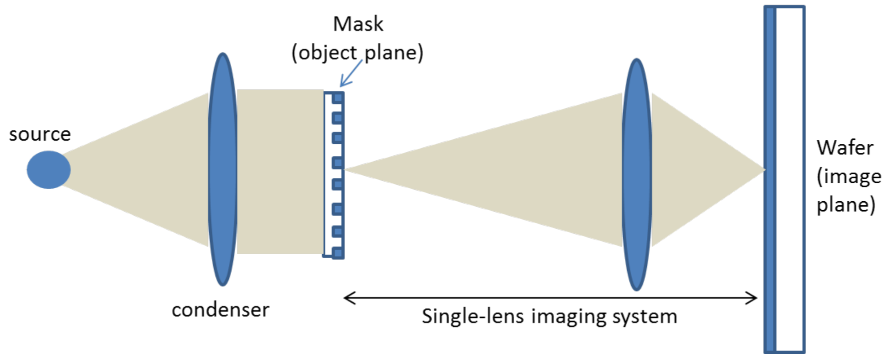


Figure 2: Schematic diagram for a projection lithography imaging system.

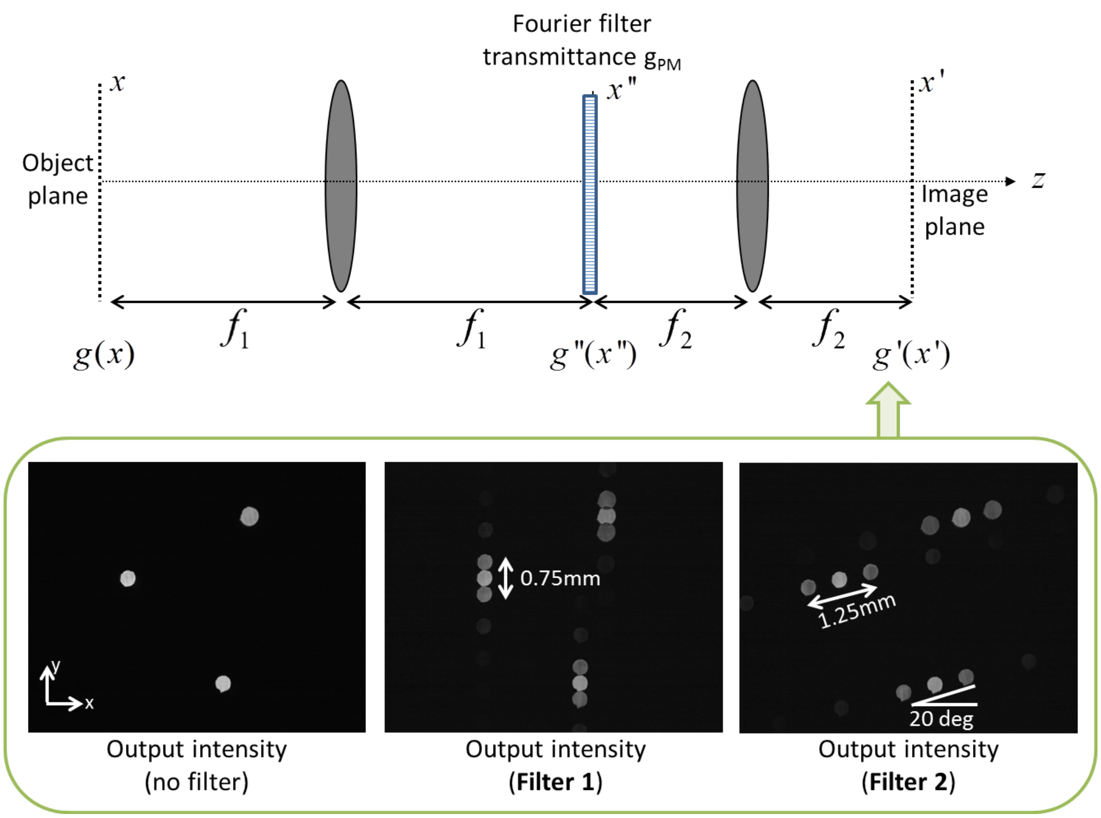
(a) To improve the resolution of the image at the wafer, should we use a longer or shorter (circle one) wavelength and larger or smaller (circle one) numerical aperture (NA)? Why?

(b) In order to print a periodic set of lines on the wafer by using a mask that is a square wave grating with pitch of 20nm and duty cycle 50%, using on-axis illumination, what is the minimum NA_{min} of the lens when using extreme ultraviolet ($\lambda = 13.5$nm) as the light source?

(c) What is the intensity distribution on the wafer for the mask and NA from part (b)?

(d) Would using a tilted plane wave illumination help to print narrower lines? Could a grating with larger pitch be printed if the frequency orders are shifted to only pass 2 diffraction orders into the lens for a fixed NA? What is the smallest line pitch of intensity you can print with the NA from part (b)?

5. **(12 points) 4*f* system with Fourier filtering.** In class, we demonstrated a home-built $4f$ system with following parameters: $f_1 = 125$mm, $f_2 = 100$mm, $\lambda = 633$nm. All marked dimensions on the images are at the image plane.



(a) What is the magnification of this imaging system? Is the image on the sensor larger or smaller than the object?

(b) Given an object that consists of three circular holes of width a, placed at positions (x_1, y_1),(x_2, y_2),(x_3, y_3), write an expression for the complex field just after the object, $g(x, y)$, when illuminated by an on-axis plane wave.

(c) Next, write an expression for the complex field,$g''(x'', y'')$, just before the Fourier filter.

(d) Next, write a general expression for the intensity,$I(x', y')$, at the image plane.

(e) We would like to guess the Fourier filter transmittance functions, $g_{PM1}(x'', y'')$ and $g_{PM2}(x'', y'')$, for each of two different filters placed in the Fourier plane. The output intensity at the image plane is shown in Fig. 5 for the case of no filter, as well as for each of two different Fourier filters (Filter 1 and Filter 2), both of which are gratings. Can you determine whether the gratings are sinusoidal, square waves, or another type? Are they 1D or 2D gratings? What is their orientation in the (x, y) plane? Which one has larger grating pitch?

(f) Assuming that the grating functions are described by the general transmittance function $g_{PM}(x'', y'') = 1 + cos(u_x x'')cos(u_y y'')$, estimate u_x and u_y for Filter 1, given the spacings and angles of the circles shown in the figure.

(g) Estimate u_x and u_y for Filter 2.