UNIVERSITY OF CALIFORNIA, BERKELEY
College of Engineering
Department of Electrical Engineering and Computer Sciences

EE 105: Microelectronic Devices and Circuits — Spring 2008

MIDTERM EXAMINATION #2
Time allotted: 80 minutes

NAME: ___**SOLUTIONS**___ ____________ ____________
(print) Last First Signature

STUDENT ID#: ____________

INSTRUCTIONS:

1. **Use the values of physical constants provided below.**
2. **SHOW YOUR WORK. (Make your methods clear to the grader!)**
3. **Clearly mark (underline or box) your answers.**
4. **Specify the units on answers whenever appropriate.**

PHYSICAL CONSTANTS

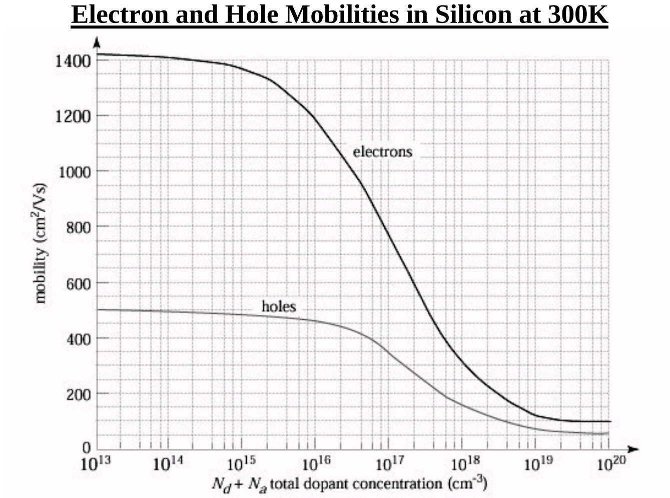
Description	Symbol	Value
Electronic charge	q	1.6×10^{-19} C
Boltzmann's constant	k	8.62×10^{-5} eV/K
Thermal voltage at 300K	$V_T = kT/q$	0.026 V

Note that $V_T \ln(10) = 0.060$ **V** at T=300K

PROPERTIES OF SILICON AT 300K

Description	Symbol	Value
Band gap energy	E_G	1.12 eV
Intrinsic carrier concentration	n_i	10^{10} cm^{-3}
Dielectric permittivity	ε_{Si}	1.0×10^{-12} F/cm

Electron and Hole Mobilities in Silicon at 300K

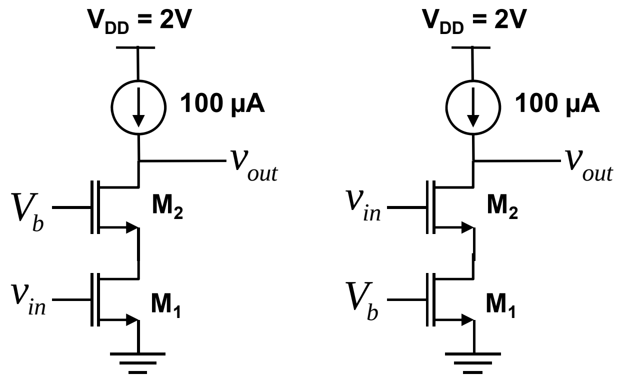


SCORE:

1	/ 20
2	/ 25
3	/ 15
4	/ 20
Total:	/ 80

<u>Problem 1</u> [20 points]: MOS Amplifiers

1) For this problem, use the following parameters for all NMOS transistors:
$V_{TH} = 0.4$ V, $\mu_n C_{ox} = 200\ \mu A/V^2$, $\lambda = 0.1 V^{-1}$, $(W/L)_1 = (W/L)_2 = 10$. The current source is ideal.



Amplifier-A

Amplifier-B

a) Find the small signal parameters for M_1 and M_2 (g_m, r_0). [4pts]

Since the amplifiers are biased identically and the MOSFETs are identical, the small-signal parameters for Amplifier-A will be the same as those for Amplifier-B.

$$g_{m1,A} = g_{m1,B} = g_{m2,A} = g_{m2,B} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_D} = \boxed{632\ \mu S}$$

$$r_{o1,A} = r_{o1,B} = r_{o2,A} = r_{o2,B} = \frac{1}{\lambda I_D} = \boxed{100\ \text{k}\Omega}$$

b) What is the topology of "Amplifier-A" (i.e., common source, common base, etc)? [2pts]

$\boxed{\text{Cascode}}$

c) Find the voltage gain, input and output resistance of Amplifier-A. Show both the expressions and the numerical values. You can make approximations in your expression as long as they are within 10% accuracy. [6pts]

$$A_v = -g_{m1}[r_{o2} + (1 + g_{m2}r_{o2})r_{o1}] = \boxed{-4126.5}$$
$$R_{in} = \boxed{\infty}$$
$$R_{out} = r_{o2} + (1 + g_{m2}r_{o2})r_{o1} = \boxed{6.525\ \text{M}\Omega}$$

d) What is the topology of "Amplifier-B" (i.e., common source, common base, etc)? [2pts]

$\boxed{\text{Common source (with degeneration)}}$

e) Find the voltage gain, input and output resistance of Amplifier-B. Show both the expressions and the numerical values. You can make approximations in your expression as long as they are within 10% accuracy. [6pts]

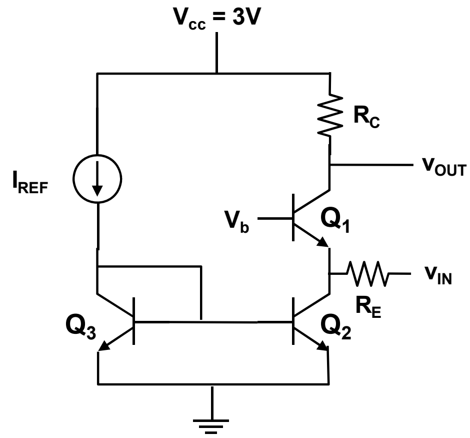
$$A_v = -\frac{g_{m2}}{1 + g_{m2}r_{o1}}[r_{o2} + (1 + g_{m2}r_{o2})r_{o1}] = \boxed{-64.23}$$

$$R_{in} = \boxed{\infty}$$

$$R_{out} = r_{o2} + (1 + g_{m2}r_{o2})r_{o1} = \boxed{6.525\ \text{M}\Omega}$$

Problem 2 [25 points]: BJT Circuits and Frequency Response

2) Assume that $V_{CC} = 3$ V, $I_{REF} = 100\mu$A, $I_S = 10^{-17}$ A, $V_A = 50$ V, and $\beta = 100$ for all transistors. $A_{E1} = A_{E2} = 10A_{E3}$. $R_C = 1$ kΩ, and $R_E = 1$ kΩ. Assume $C\mu = 10$ fF, $C\pi = 100$ fF, and $C_{CS} = 20$ fF.



a) Identify the functions of Q_1, Q_2, and Q_3. What is the function of this circuit? [3pts]

Q_1 is the amplifying transistor. Q_2 provides a current bias for Q_1 by mirroring the current from Q_3. Q_3 provides a reference current by biasing the base of Q_2. The circuit is a common-base voltage amplifier.

b) Find the small-signal parameters of the main amplifier transistor (r_π, g_m, r_0). [3pts]
Note $\beta \gg 1$.

$$I_{C1} = \frac{A_{E2}}{A_{E3}} I_{REF} = 1 \text{ mA}$$

$$g_{m1} = \frac{I_{C1}}{V_T} = \boxed{\frac{1}{26} \text{ S}}$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \boxed{2.6 \text{ k}\Omega}$$

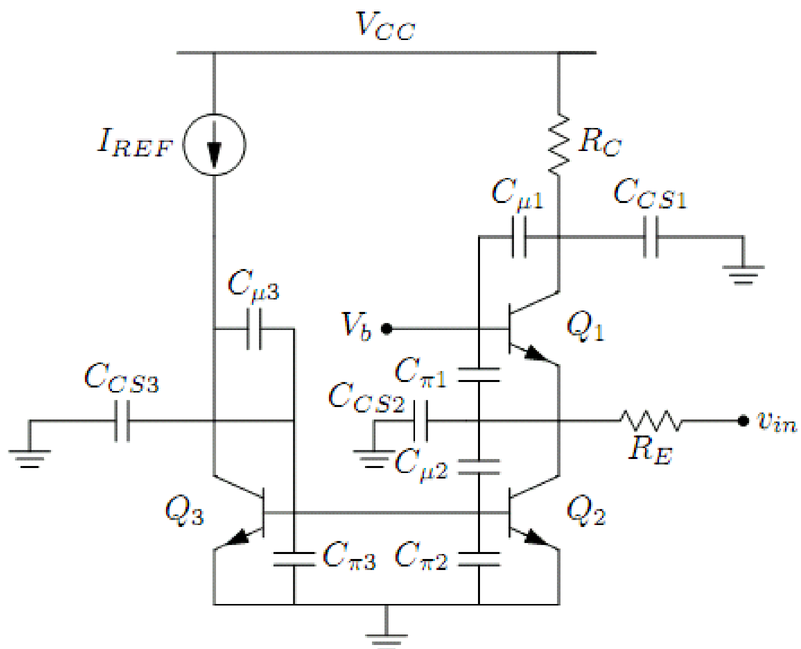
$$r_{o1} = \frac{V_A}{I_C} = \boxed{50 \text{ k}\Omega}$$

c) Find the <u>expression</u> and the <u>value</u> of the voltage gain. [4pts]
Note $R_C \ll r_o$.

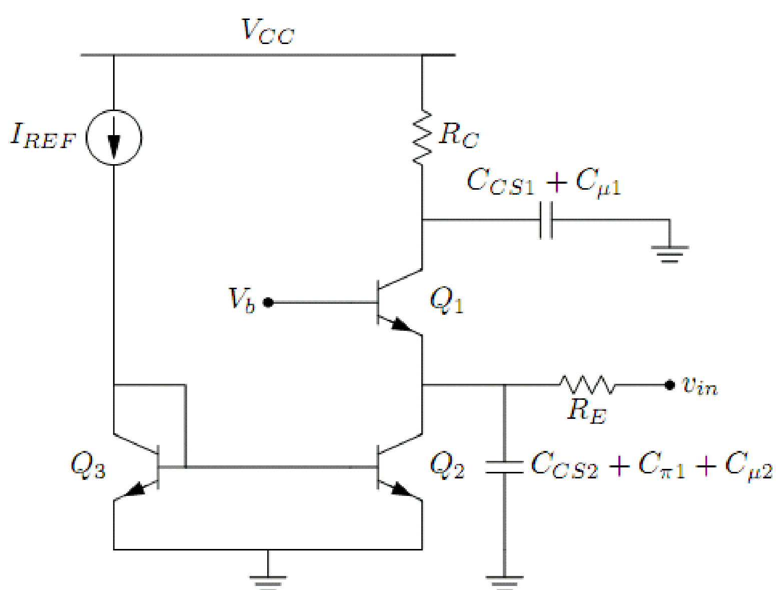
$$A_v = \boxed{\frac{\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{o2}}{R_E + \frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{o2}} g_{m1} R_C} \approx \frac{\frac{1}{g_{m1}}}{R_E + \frac{1}{g_{m1}}} g_{m1} R_C \approx \frac{R_C}{R_E + \frac{1}{g_{m1}}} = \boxed{0.975}$$

d) Show all parasitic capacitances of the BJTs in the circuit diagram above. Simplify the capacitances (e.g., combine all capacitances in parallel, remove capacitances that are shorted). Redraw the circuit diagram below with the simplified capacitances. [Hint: a constant DC voltage is AC ground]. [5pts]

Here's the circuit with all parasitic capacitances:



And the one with the capacitances simplified:



e) Find the input and output poles of this circuits. What is the dominant pole? Find the 3-dB bandwidth of this circuit. [5pts].

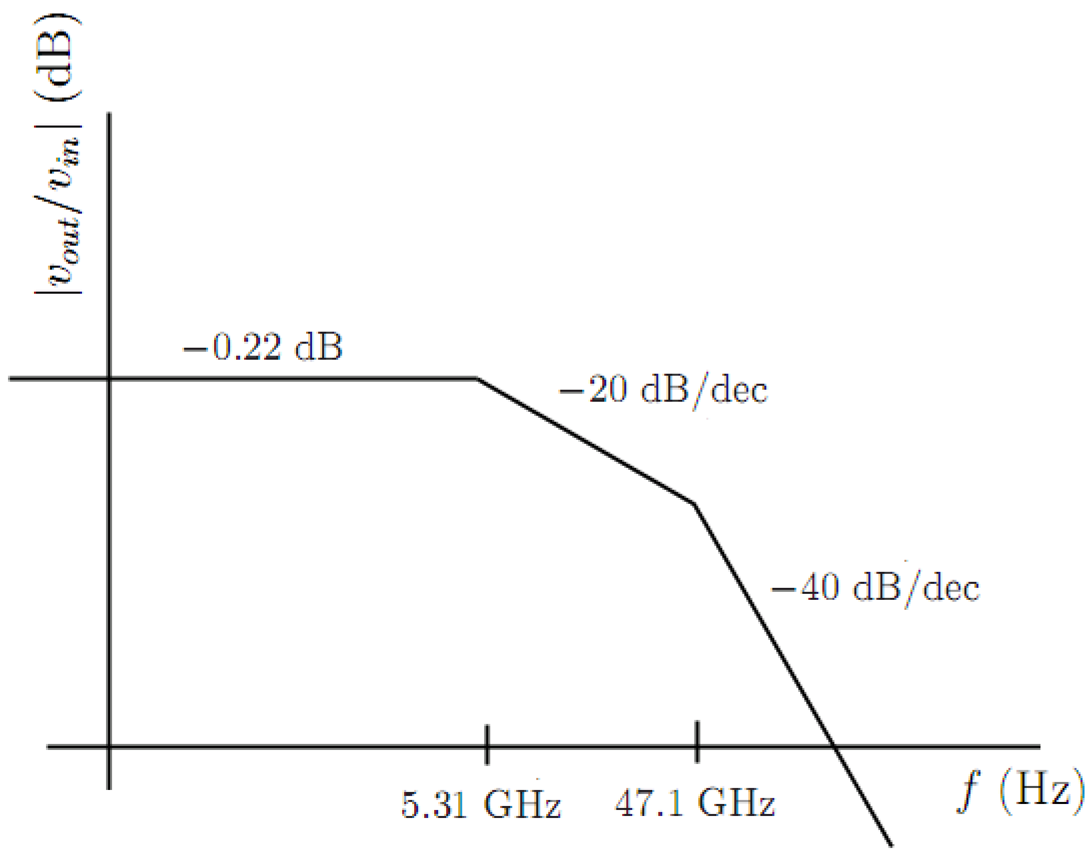
$$\omega_{p,in} = \frac{1}{\left(R_E \parallel \frac{1}{g_{m1}} \parallel r_{o2}\right)(C_{CS2} + C_{\mu 2} + C_{\pi 1})} \approx \frac{1}{\frac{1}{g_{m1}}(C_{CS2} + C_{\mu 2} + C_{\pi 1})}$$
$$= 2.96 \times 10^{11}\ \text{rad/s} = \boxed{47.1\ \text{GHz}}$$

$$\omega_{p,out} = \frac{1}{R_C(C_{\mu 1} + C_{CS1})} = 3.33 \times 10^{10}\ \text{rad/s} = \boxed{5.31\ \text{GHz}}$$

The dominant pole is the smaller pole, so it is $\boxed{5.31\ \text{GHz}}$.

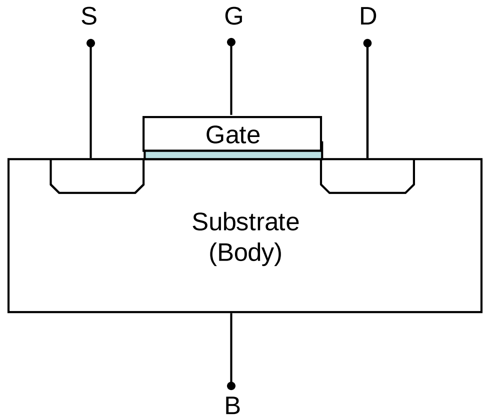
f) Construct the Bode plot of the transistor. Clearly mark the scale of both axes. The Bode plot should show both the low-frequency voltage gain as well as 3-dB bandwidth of the amplifier. [5pts]

The low frequency gain on the plot is $20 \log 0.975 = -0.22$ dB.



<u>Problem 3</u> [15 points]: MOS Devices

3) Below is the cross section of a PMOS transistor:



a) What is the doping type (n, n+, p, or p+, where "+" means high doping concentration) of [3pts]

i) Source: $\boxed{\text{p+}}$

ii) Drain: $\boxed{\text{p+}}$

iii) Substrate (body): $\boxed{\text{n}}$

b) Which carrier(s) are involved in current conduction? (i) electrons, (ii) holes), (iii) both electrons and holes. (choose one) [3pts]

$\boxed{\text{Holes}}$

c) If the power supply voltage is 2V and ground is 0V, what bias voltage is usually connected to the body (substrate) of the transistor? Why? [3pts]

$\boxed{\text{2 V}}$

We want to ensure the n-type side of the pn junctions is at the highest potential to keep them reverse biased. Otherwise, we'll get substrate current (undesirable).

d) Assume the threshold voltage of the PMOS is V_{TH} = -0.4V. If V_S = 2V, V_D = 0V, find the gate voltages for which the PMOS is (i) cut-off, (ii) in between saturation and triode regions. [3pts]

(i) $V_{GS} > V_{TH} \Rightarrow V_G > V_S + V_{TH} \Rightarrow \boxed{V_G > 1.6\text{ V}}$

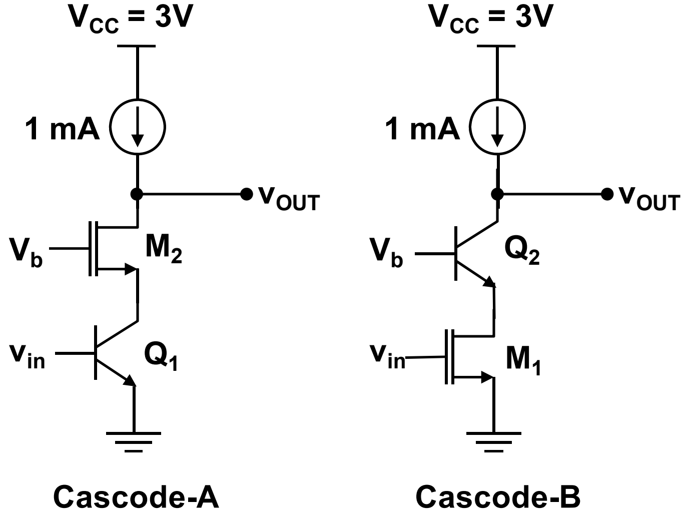
(ii) $V_{DS} = V_{GS} - V_{TH} \Rightarrow V_G = V_D + V_{TH} \Rightarrow \boxed{V_G = -0.4\text{ V}}$

e) If a PMOS and an NMOS have exactly the same dimension (W, L, and oxide thickness), which transistor has higher g_m? Why? (Assume both are long-channel devices) [3pts]

The $\boxed{\text{NMOS}}$ will have higher transconductance because electrons have higher mobility than holes and $g_m \propto \sqrt{\mu}$.

Problem 4 [20 points]: Cascode Amplifiers

4) Below is a cascode amplifier with *mixed* MOS and BJT transistors:



MOS transistor parameters: $V_{TH} = 0.5V$, $\mu_nC_{ox} = 200\ \mu A/V^2$, $\lambda = 0.1V^{-1}$, $(W/L)_1 = (W/L)_2 = 10$

BJT transistor parameters: $I_S = 10^{-17}$ A, $V_A = 10$ V, and $\beta = 100$

a) Find the small-signal parameters of the MOS transistor (g_m, r_0). [2pts]

Since they're biased identically in each circuit (once again, using the approximation that $\beta \gg 1$) the MOSFETs will have identical small-signal parameters.

$$g_{m2,A} = g_{m1,B} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D} = \boxed{2\text{ mS}}$$

$$r_{o2,A} = r_{o1,B} = \frac{1}{\lambda I_D} = \boxed{10\text{ k}\Omega}$$

b) Find the small-signal parameters of the BJT transistor (g_m, r_0, r_π). [3pts]

Since they're biased identically in each circuit (once again, using the approximation that $\beta \gg 1$) the BJTs will have identical small-signal parameters.

$$g_{m1,A} = g_{m2,B} = \frac{I_C}{V_T} = \boxed{\frac{1}{26}\text{ S}}$$

$$r_{\pi1,A} = r_{\pi2,B} = \frac{\beta}{g_m} = \boxed{2.6\text{ k}\Omega}$$

$$r_{o1,A} = r_{o2,B} = \frac{V_A}{I_C} = \boxed{10\text{ k}\Omega}$$

c) From **a)** and **b)**, find the *relative* magnitudes of small-signal parameters: [3pts]

i) $g_{m,BJT} \,/\, g_{m,MOS}$

ii) $r_{0,BJT} \,/\, r_{0,MOS}$

iii) $r_{\pi,BJT} \,/\, r_{0,BJT}$

(i) $\frac{g_{m,BJT}}{g_{m,MOS}} = \boxed{19.23}$

(ii) $\frac{r_{o,BJT}}{r_{o,MOS}} = \boxed{1}$

(iii) $\frac{r_{\pi,BJT}}{r_{o,BJT}} = \boxed{0.26}$

d) Derive the expressions of the output resistances for both cascade amplifiers shown above. You can drop small terms that are less than 10% of the dominant terms. Using the ratios you obtained in **c)**, determine which cascode amplifier (Cascode-A or Cascode-B) has higher output resistance. [6pts]

$$R_{out,A} = \boxed{r_{o2} + (1 + g_{m2}r_{o2})r_{o1}} \approx g_{m2}r_{o2}r_{o1}$$

$$R_{out,B} = \boxed{r_{o2} + (1 + g_{m2}r_{o2})(r_{o1} \parallel r_{\pi 2})} \approx g_{m2}r_{o2}(r_{o1} \parallel r_{\pi 2})$$

$$\frac{R_{out,A}}{R_{out,B}} \approx \frac{g_{m,MOS}r_{o,MOS}\, r_{o,BJT}}{g_{m,BJT}r_{o,BJT}(r_{o,MOS} \parallel r_{\pi,BJT})} = \frac{1}{19.23} \cdot 1 \cdot \frac{r_{o,BJT}}{(r_{o,MOS} \parallel r_{\pi,BJT})} < 1$$

Thus, $\boxed{\text{Cascode-B}}$ has higher output resistance.

e) Derive the expressions of the voltage gain for both cascade amplifiers shown above. You can drop small terms that are less than 10% of the dominant terms. Using the ratios you obtained in **c)**, determine which cascode amplifier (Cascode-A or Cascode-B) has higher voltage gain. [6pts]

$$A_{v,A} = \boxed{-g_{m1}[r_{o2} + (1 + g_{m2}r_{o2})r_{o1}]} \approx -g_{m1}g_{m2}r_{o2}r_{o1}$$

$$A_{v,B} = \boxed{-g_{m1}[r_{o2} + (1 + g_{m2}r_{o2})(r_{o1} \parallel r_{\pi 2})]} \approx -g_{m1}g_{m2}r_{o2}(r_{o1} \parallel r_{\pi 2})$$

$$\frac{A_{v,A}}{A_{v,B}} \approx \frac{g_{m,BJT}\,g_{m,MOS}\,r_{o,MOS}\,r_{o,BJT}}{g_{m,MOS}\,g_{m,BJT}\,r_{o,BJT}(r_{o,MOS} \parallel r_{\pi,BJT})} = \frac{r_{o,BJT}}{(r_{o,MOS} \parallel r_{\pi,BJT})} > 1$$

Thus, $\boxed{\text{Cascode-A}}$ has higher gain.