University of California at Berkeley
College of Engineering
Dept. of Electrical Engineering and Computer Sciences

EE 105 Midterm II

Spring 2002 | Prof. Roger T. Howe | April 17, 2002

Solutions,
Your Name (Last, First)

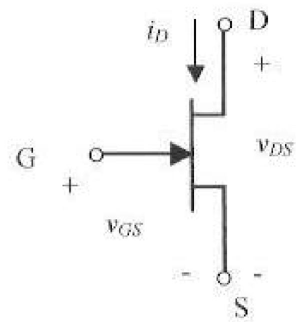
Guidelines

Closed book and notes; one 8.5" x 11" page (both sides) of *your own notes* is allowed.
You may use a calculator.
Do not unstaple the exam.
Show all your work and reasoning on the exam in order to receive full or partial credit.

Score

Problem	Points Possible	Score
1	16	
2	18	
3	16	
Total	50	

1. Junction Field-Effect Transistor (JFET) Model. [16 points].



Device parameters:

I_{DSS} = 125 μA
V_P = - 1.5 V
λ_n = 0.05 V^{-1}

A simplified large-signal model for an n-channel JFET is:

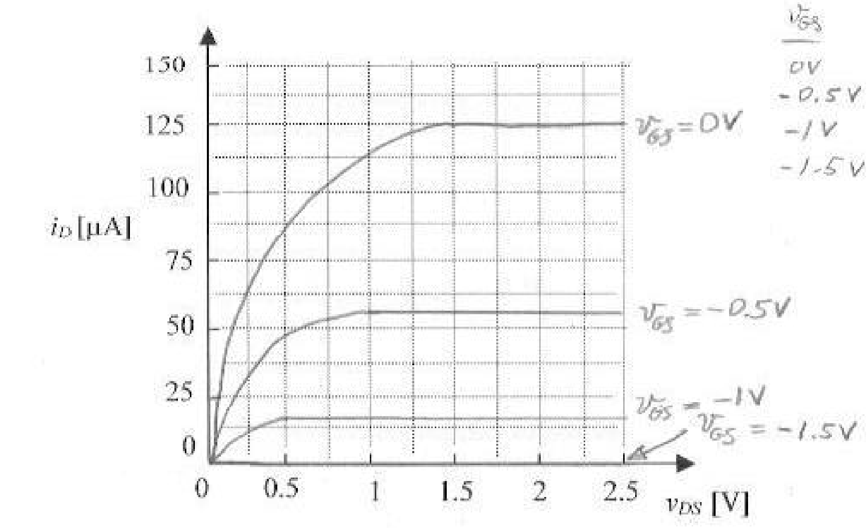
$$i_D = \frac{2I_{DSS}}{V_P^2}(v_{GS} - V_P - \frac{v_{DS}}{2})v_{DS}(1+\lambda_n v_{DS}) \text{ for } v_{DS} \le v_{GS} - V_P \text{ and } V_P \le v_{GS} \le 0\text{ V (triode)}$$

$$i_{D_{SAT}} = \frac{I_{DSS}}{V_P^2}(v_{GS} - V_P)^2(1+\lambda_n v_{DS}) \text{ for } v_{DS} \ge v_{GS} - V_P \text{ and } V_P \le v_{GS} \le 0\text{ V (saturation)}$$

where V_P is the pinch-off voltage and λ_n is the "fudge factor."

(a) [4 pts.] Sketch the drain characteristics for this JFET on the graph below for V_{GS} = 0 V, -0.5 V, -1 V, and –1.5 V. You can set λ_n= 0 for this part. Your current values in saturation should be accurate.

v_{GS}	$i_{D_{SAT}}$ (μA)	$v_{DS_{SAT}}$
0V	125μA	1.5V
-0.5V	55	1V
-1V	14	0.5
-1.5V	0	0



(b) [4 pts.] What is the numerical value of the small-signal transconductance g_m at the operating point Q_1 (V_{GS} = -0.5 V, V_{DS} = 1.5 V)? Notes: (i) λ_n is not zero for this part, (ii) you don't need the plots in part (a) in order to answer this question.

$$g_{m_1} = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Q_1}$$

Q_1 is in saturation since $V_{DS} = 1.5V > V_{GS} - V_P = -0.5V - (-1.5V) = 1V$

$$i_D = \frac{I_{DSS}}{V_P^2}\left(v_{GS} - V_P\right)^2\left(1 + \lambda_n v_{DS}\right)$$

$$g_m = \left(\frac{2I_{DSS}}{V_P^2}\right)\left(V_{GS} - V_P\right)\left(1 + \lambda_n V_{DS}\right)$$

$$g_m = \left[\frac{2(125\mu A)}{(-1.5V)^2}\right]\left(-0.5V - (-1.5V)\right)\left(1 + (0.05V^{-1})(1.5V)\right)$$

$$\boxed{g_m = 119 \mu S}$$

(c) [4 pts.] What is the numerical value of the small-signal drain resistance r_o at the operating point Q_1 (V_{GS} = -0.5 V, V_{DS} = 1.5 V). Notes: (i) λ_n is not zero for this part, (ii) you don't need the plots in part (a) in order to answer this question.

$$r_o^{-1} = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_{Q_1} = \left(\frac{I_{DSS}}{V_P^2}\right)\left(V_{GS} - V_P\right)^2 \lambda_n$$

$$= \frac{125\mu A}{(-1.5V)^2}\left(-0.5V - (-1.5V)\right)\left(0.05 V^{-1}\right)$$

$$r_o^{-1} = 2.78 \mu S \Rightarrow \boxed{r_o = 360 k\Omega}$$

(d) [4 pts.] What is the numerical value of the small-signal transconductance g_m at the operating point Q_2 (V_{GS} = -0.5 V, V_{DS} = 0.5 V). Again, you don't need the plot in part (a) in order to answer this question.

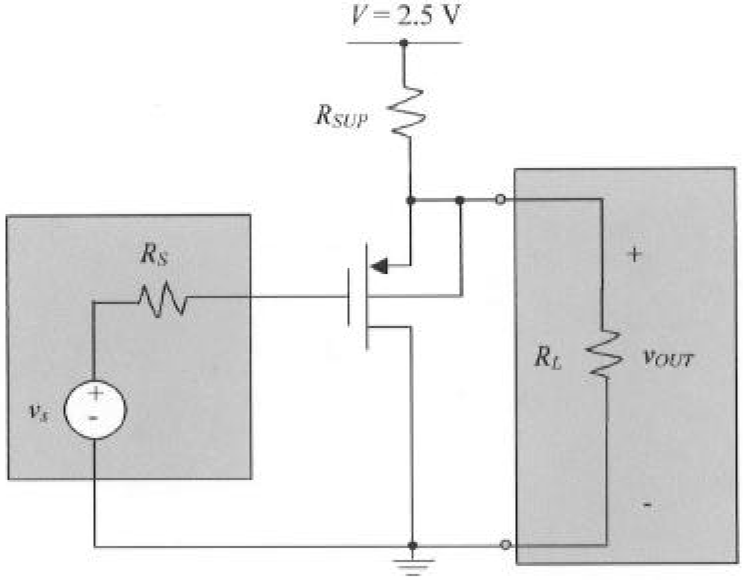
$V_{DS} = 0.5V < V_{GS} - V_P = -0.5V - (-1.5V) = 1V \Rightarrow$ Triode region

$$g_{m_2} = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Q_2} = \left(\frac{2I_{DSS}}{V_P^2}\right) V_{DS}\left(1 + \lambda_n V_{DS}\right)$$

$$= \left(\frac{250\mu A}{(-1.5V)^2}\right)\left(0.5V\right)\left(1 + (0.05V^{-1})(0.5V)\right)$$

$$\boxed{g_{m_2} = 57 \mu S}$$

2. MOSFET single stage amplifier [18 pts.]



Given:
$L = 1\ \mu m$
$\mu_p C_{ox} = 62.5\ \mu A/V^2$
$V_{Tp} = -1$ V
$\lambda_p = 0.025\ V^{-1}$
$g_{mb} = g_m/10$

$R_S = 10\ k\Omega$
$R_{SUP} = 5\ k\Omega$
$R_L = 1\ k\Omega$

(a) [3 pts.] Find the numerical value of channel width W in μm in order that the DC output voltage $V_{OUT} = 1.25$ V. *Note*: the gray boxes indicate small-signal elements that can be neglected for the DC bias analysis.

$$V_{SG} = V_{OUT} - V_G = 1.25V - 0V = 1.25V$$

$$-I_{Dp} = I_{R_{SUP}} = \frac{2.5V - 1.25V}{5k\Omega} = 250\mu A$$

$$-I_{Dp} = \mu_p C_{ox}(W/2L)(V_{SG} + V_{Tp})^2 \Rightarrow W = \frac{2L(-I_{Dp})}{\mu_p C_{ox}(V_{SG} + V_{Tp})^2}$$

$$= \frac{500\mu A}{(62.5\mu A/V^2)(1.25-1)^2}$$

$$\boxed{W = 128\mu m}$$

(b) [3 pts.] What is DC power dissipated in the MOSFET in μW?

$$P = -I_{Dp} \cdot V_{SD} = (250\mu A)(1.25V) = \boxed{312.5\mu W}$$

(c) [3 pts.] Find the numerical value of the output resistance R_{out} of this amplifier in kΩ. If you couldn't solve part (a), you can assume for this part that the channel width $W = 100$ μm (not the correct answer to (a), of course.)

$$R_{out} = \frac{1}{g_m} \| R_{sup} \quad (v_{sb} = 0 \Rightarrow \text{ignore } g_{mb} \text{ generator})$$

$$\frac{1}{g_m} = \frac{1}{\mu_p C_{ox}(W/L)(V_{SG} + V_{Tp})} = \frac{1}{(62.5\mu A/V^2)(128)(1.25-1)} = 500\Omega$$

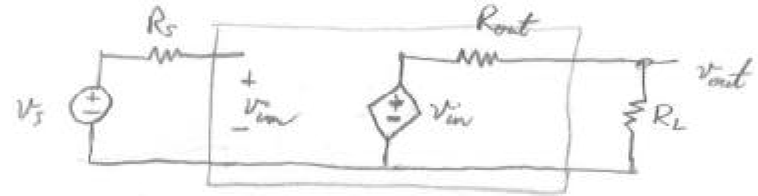
$$R_{out} = 500\Omega \| 5k\Omega$$

$$\boxed{R_{out} = 454\Omega}$$

(d) [3 pts.] Find the numerical value of the two-port parameter A_v, the open-circuit voltage gain, for this amplifier. Again, if you couldn't solve part (a), you can assume for this part that the channel width $W = 100$ μm (not the correct answer to (a), of course.)

$\boxed{A_v = 1}$ since g_{mb} can be ignored.

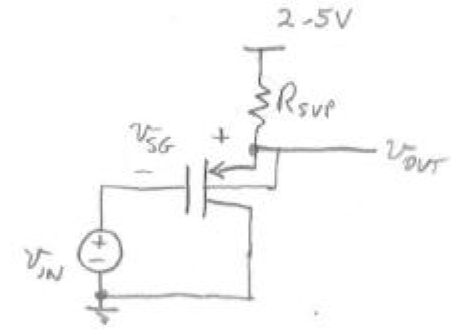
(e) [3 pts.] Find the overall voltage gain v_{out} / v_s with R_S and R_L present (values of which are given next to the schematic on the previous page). If you couldn't solve (c) or (d), you can assume for this part that $R_{out} = 2.5$ kΩ, and $A_v = 0.85$. Needless to say, these are not correct answers to either (c) or (d).



$$v_{out}/v_S = \frac{R_L}{R_{out} + R_L} = \frac{1000\Omega}{454\Omega + 1000\Omega}$$

$$\boxed{v_{out}/v_S = 0.69}$$

(f) [3 pts.] We now remove the small-signal source and its resistance and replace it with a large-signal source v_{IN}; we also remove the load resistor. Assuming the MOSFET remains in the saturation (constant-current) region, find an equation for v_{IN} in terms of v_{OUT}. What is the numerical value of v_{IN} for the case when $v_{OUT} = 2$ V? If you couldn't solve part (a), you can assume that $W = 100$ μm for this part.



$$v_{OUT} = 2.5V - (-i_{Dp}) R_{SUP}$$

$$-i_{Dp} = \mu_p C_{ox} (W/2L)(v_{SG} + V_{Tp})^2 (1 + \lambda_n v_{SD})$$

$$v_{SG} = v_{OUT} - v_{IN}$$

$$v_{SD} = v_{OUT}$$

$$-i_{Dp} = \mu_p C_{ox} (W/2L)(v_{OUT} - v_{IN} + V_{Tp})^2 (1 + \lambda_n v_{OUT})$$

$$v_{OUT} = 2.5V - \mu_p C_{ox} R_{SUP} (W/2L)(v_{OUT} - v_{IN} + V_{Tp})^2 (1 + \lambda_n v_{OUT})$$

$$(\mu_p C_{ox} R_{SUP} (W/2L)(1 + \lambda_n v_{OUT}))(v_{OUT} - v_{IN} + V_{Tp})^2 = 2.5V - v_{OUT}$$

$$v_{OUT} - v_{IN} + V_{Tp} = \sqrt{\frac{2.5V - v_{OUT}}{\mu_p C_{ox} R_{SUP} (\frac{W}{2L})(1 + \lambda_n v_{OUT})}}$$

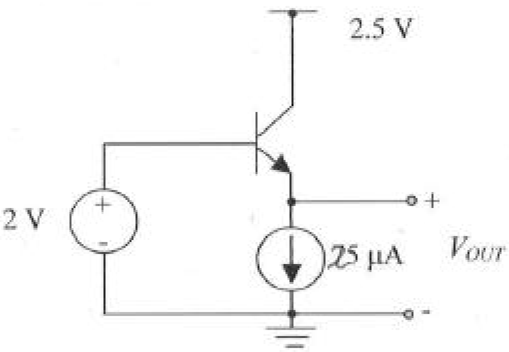
$$\boxed{v_{IN} = v_{OUT} + V_{Tp} - \sqrt{\frac{2.5V - v_{OUT}}{\mu_p C_{ox} R_{SUP} (\frac{W}{2L})(1 + \lambda_n v_{OUT})}}}$$

$$v_{OUT} = 2V \Rightarrow \quad v_{IN} = (2V - 1V) - \sqrt{\frac{2.5V - 2V}{(62.5\times10^{-3})(5)(\frac{128}{2})(1 + 0.05(2))}}$$

$$\boxed{v_{IN} = 0.85V}$$

neglecting $\lambda_n \Rightarrow \underline{v_{IN} \approx 0.84V}$

3. npn bipolar transistors [16 pts.]



Given:
Base width = W_B = 100 nm = 0.1 μm
Emitter-base junction area = $A_E = \cancel{1}.5$ μm^2
Emitter width = W_E = 75 nm = 0.075 μm
Base-collector junction area = A_C = 15 μm^2
Electron diffusion constant in base: $D_n = 10$ cm^2/s
Hole diffusion constant in emitter: $D_p = 5$ cm^2/s
Electron charge: $q = -1.6 \times 10^{-19}$ C
Intrinsic concentration: $n_i = 10^{10}$ cm^{-3}
V_{th} = 26 mV

(a) [4 pts.] Find the numerical value of the electron diffusion current density J_{nB} in the base [units μA/μm^2]. Neglect the base current I_B for this part.

$$I_C = J_{nB} A_E \qquad I_B = 0 \Rightarrow I_C = -I_E = 25\,\mu A$$

$$J_{nB} = \frac{I_C}{A_E} = \frac{25\,\mu A}{5\,\mu m^2} = \boxed{5\ \mu A/\mu m^2}$$

(b) [4 pts.] What is the numerical value of $n_{pB}(x = 0)$, the minority electron concentration in the base at the edge of the emitter-base depletion region? Again, you can neglect the base current I_B for this part.

$$J_{nB} = \frac{q D_{nB}\, n_{pB}(x=0)}{W_B} \Rightarrow n_{pB}(x=0) = \frac{J_{nB} W_B}{q \cdot D_{nB}}$$

$$= \frac{(5\,\mu A/\mu m^2)(10^{8}\,cm^2/\mu m^2)(10^{-5}\,cm)}{(1.6\times10^{-19}\,C)(20\,cm^2/s)}$$

$$\boxed{n_{pB}(0) = 1.56 \times 10^{15}\,cm^{-3}}$$

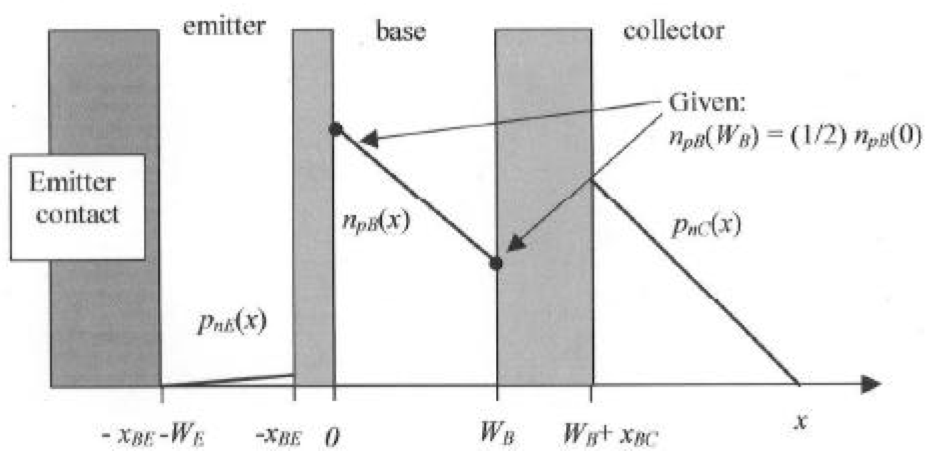
(c) [3 pts.] Find the numerical value of V_{OUT} to 3 significant figures. The base doping is $N_{aB} = 10^{17}$ cm^{-3}. You can neglect the base current for this part, too.

$$n_{pB}(x=0) = n_{pBo} e^{V_{BE}/V_{th}} \Rightarrow V_{BE} = V_{th} \ln\left(\frac{n_{pB}(x=0)}{n_{pBo}}\right) = 26mV \ln\left[\frac{1.56\times10^{15}}{1000}\right]$$

$$n_{pBo} = \frac{n_i^2}{N_{aB}} = \frac{10^{20}}{1\times10^{17}} = 10^3 cm^{-3} \qquad = 730.0 mV$$

$$V_{OUT} = V_B - V_{BE} = 2 - 0.730 V = \boxed{1.270 V}$$

(d) [4 pts.] We now increase V_B above 2 V to the point where the minority carrier concentrations in the bipolar transistor are given by the plot below. The value of $n_{pB}(0)$ is unchanged from parts (b) and (c). What is the value of V_B to 3 significant figures? *Note*: if you can't find the exact value, the answer to 2 significant figures is worth 2 pts.



BJT is saturated $\Rightarrow V_{CE_{sat}} \cong 0.1 V \Rightarrow V_{OUT} \approx 2.4V, \; V_B \cong 3.1V$

Exact value: find V_{BC} from law of the junction

$$n_{pB}(x=W_B) = n_{pBo} e^{V_{BC}/V_{th}}$$

$$V_{BC} = V_{th} \ln\left(\frac{n_{pB}(W_B)}{n_{pBo}}\right) = 26mV \ln\left(\frac{\frac{1.56}{2}\times10^{15} cm^{-3}}{1000 cm^{-3}}\right)$$

$$= 712mV$$

$$\therefore V_{CE} = V_{CB} + V_{BE} = -V_{BC} + V_{BE}$$

$$= -712mV + 730mV$$

$$V_{CE} = 18mV$$

$$\therefore V_B = 2.5 - V_{CB} = 2.5V - (-0.712V)$$

$$\boxed{V_B = 3.21 V}$$