Solution

# UNIVERSITY OF CALIFORNIA, BERKELEY
## College of Engineering
## Department of Electrical Engineering and Computer Sciences

EE 105: Microelectronic Devices and Circuits

Fall 2009

# MIDTERM EXAMINATION #1

Time allotted: 45 minutes

**NAME:** ______________________ ______________________

**STUDENT ID#:** ____________________

**INSTRUCTIONS:**

1. **SHOW YOUR WORK. (Make your methods clear to the grader!)** Specially, while using chart, make sure that you indicate how you have got your numbers. For example, if reading off mobility, clearly write down what doping density that corresponds to.
2. **Clearly mark (underline or box) your answers.**
3. **Specify the units on answers whenever appropriate.**

**SCORE: 1** __________ **/ 15**

**2** __________ **/ 15**

**3** __________ **/ 20**

**Total** __________ **/ 50**

## PHYSICAL CONSTANTS

| Description | Symbol | Value |
|---|---|---|
| Electronic charge | $q$ | $1.6\times10^{-19}$ C |
| Boltzmann's constant | $k$ | $8.62\times10^{-5}$ eV/K |
| Thermal voltage at 300K | $V_T = kT/q$ | 0.026 V |

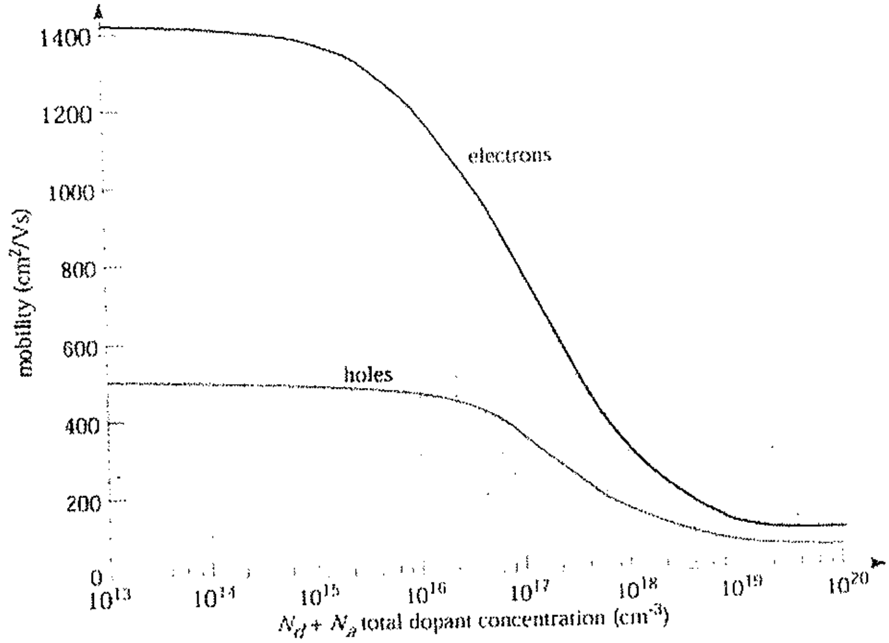
## PROPERTIES OF SILICON AT 300K

| Description | Symbol | Value |
|---|---|---|
| Band gap energy | $E_G$ | 1.12 eV |
| Intrinsic carrier concentration | $n_i$ | $10^{10}$ cm$^{-3}$ |
| Dielectric permittivity | $\varepsilon_{Si}$ | $1.0\times10^{-12}$ F/cm |

## USEFUL NUMBERS

**$V_T \ln(10) = 0.060$ V** at $T$=300K
**$\exp(30) \sim 10^{13}$**

## Electron and Hole Mobilities in Silicon at 300K



**Prob 1**. **[15 pts]** Let us assume that we have a Si sample of length 16 μm and a cross sectional area of 10 $\mu m^2$ at room temperature. Also assume that we apply 1V across this sample.

(a) If it is desired to have an electron density that is $10^5$ times the intrinsic density, [4 pt]

- (i) What kind of dopant atoms is needed? Can you give an example?
- (ii) What doping density will be required?

(i) Donors → gp V → phosphorous

(ii) $n = 10^{15}/cm^3 \approx N_D$

(b) Estimate the drift velocity of this sample. [5 pt]

$$v = \mu_n \mathcal{E} = \underset{\uparrow}{1350} \times \frac{1(V)}{16 \times 10^{-4}(cm)}$$

from chart corresponding to $10^{15}$

$$\cong 8 \times 10^5 \ cm/sec$$

(c) Now let us assume that we counter-dope the sample with opposite type of dopants with a density that is 3 times more than the previous dopant density. Estimate the resistance and current flowing in this sample under these conditions. [6 pt]

material becomes p-type : $p = (3-1) \times 10^{15} = 2 \times 10^{15}$ $N_A$ $N_D$

$$R = \rho \frac{L}{A} = \frac{1}{\sigma} \cdot \frac{L}{A}$$

$\sigma = q \mu_p p \rightarrow \mu_p \sim \underline{500}$ corresponding to $\underbrace{N_A + N_D}$ $= 4 \times 10^{15}\ cm^{-3}$
20

$$\therefore R = \frac{1}{1.6 \times 10^{-19} \times 500 \times 2 \times 10^{15}} \times \frac{16 \times 10^{-4}}{20 \times (10^{-4})^2}$$

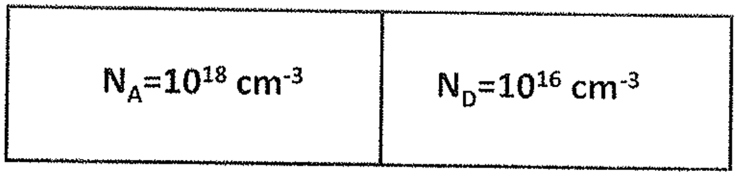
$$= \frac{1}{10^{-19} \times 10^{18} \times 10^{-4}}$$

$$\boxed{R = 10^5\ \Omega}$$

$$I = \frac{10}{10^5\ \Omega} = 10^{-5}\ A$$

$$\boxed{I = 10\ \mu A}$$

**Prob 2[15 pts].** Consider a p-n junction diode of Si as shown below:



(a) Find out the built in potential and the depletion width at T=300K. [4 pt]

$V_i = V_T \ln \frac{N_A N_D}{n_i^2}$

$= V_T \ln \frac{10^{18} \times 10^{16}}{10^{20}}$

$= 14 \times 0.06$

$\boxed{V_i = 0.84V}$

$W = \sqrt{\frac{2\epsilon}{q} \times \left(\frac{1}{N_A} + \frac{1}{N_D}\right) V_i}$

$= \sqrt{\frac{2 \times 10^{-12}}{1.6 \times 10^{19}} \times \underbrace{\frac{1}{10^{16}}}_{N_D} \times 0.84}$

$\simeq \sqrt{6.4 \times 10^{9}}$

$\boxed{W \simeq 3 \times 10^{5}\ cm}$

(b) Assume that at T=300K a voltage of 0.9 volt is applied across the diode such that the diode is forward biased. How much should the voltage have to be changed if at T=400 K one needs to ensure that the same current is flowing as in T=300K? Assume no change in bandgap due to change in temperature. [5 pt]

Same current appears when:

$V_1 - \underbrace{V_{i1}}_{\text{Built in pot}} = V_2 - \underbrace{V_{i2}}_{\text{Built in pot.}}$

$V_{i2} = V_{T_2} \ln \frac{N_A N_D}{n_i^2}$

$n_i = 5.2 \times 10^{15} \times (400)^{3/2} \times e^{-\frac{1.12}{0.026 \times 4/3}}$

$\simeq 4 \times 10^{12}$

$n_i^2 \simeq 10^{25}$

$V_{i2} = \frac{4}{3} \times V_T \ln \frac{10^{16} \times 10^{18}}{10^{25}}$

$V_{i2} = \frac{4}{3} \times \overset{3}{9} \times \underbrace{V_T \ln 10}_{0.06}$

$= 0.72$

$\therefore V_2 = V_1 - V_{i1} + V_{i2}$

$= 0.9 - 0.84 + 0.72$

$\boxed{V_2 = 0.78V}$

(c) In this problem we shall design a diode to meet certain requirements. We want a total current of 40 A/cm$^2$ at a voltage of 0.8 V at T=300K. Half of the current will have to be supplied by holes and half by electrons. Assume the mobility of electrons to be 500 cm$^2$/V-sec and that of holes to be 250 cm$^2$/V-sec. Also assume that the diffusion length for electrons and holes are the same and equal to 10 μm. Find out how you will design your diode, i.e., how you will dope your p and n sides so that the aforementioned requirements are met. [6 pt]

$$J_n \simeq \frac{e n_i^2}{L} \frac{D_n}{N_A} e^{\frac{V_D}{V_T}} = \frac{40}{2}$$

$$\simeq \frac{e n_i^2 V_T}{L} \frac{\mu_n}{N_A} e^{\frac{V_D}{V_T}} = 20$$

$$\therefore N_A = \frac{1\,6\times10^{19}\times10^{20}\times\overbrace{26\times10^{3}}^{V_T}}{\underbrace{10\times10^{4}}_{L}}\times\underbrace{500}_{\mu_n}\times\underbrace{10^{13}}_{e^{\frac{0.8}{0.026}}\simeq e^{30}\simeq\underbrace{10^{13}}_{\text{see from pg.}}}$$

$$= 26\times5\times10^{15}$$

$$\boxed{N_A = 1\,3\times10^{17}\ \text{cm}^{-3}}$$

$$J_p = \frac{e n_i^2}{L}\times\frac{\mu_p V_T}{N_D} e^{\frac{V_D}{V_T}} = J_n$$

$$\therefore \frac{N_A}{N_D} = \frac{\mu_n}{\mu_p}$$

$$\therefore \boxed{N_D = \frac{N_A}{2}}$$

**Prob 3. [20 pts]**Bipolar junction transistors.

(a) Design Fundamentals [4 pts].

(i) Why is the base region doped more lightly than the emitter? [2 pt]

increases emitter to base flow

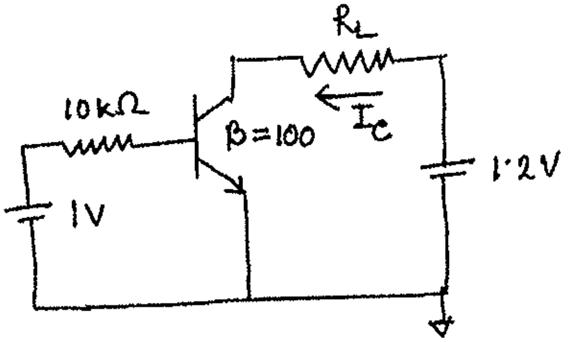
decreases base to emitter flow

hence increases gain.

(ii) Why is the width of the base region narrower than emitter and collector regions? [2 pt]

to minimize probability of recombination.

(b) Write down the condition for active mode operation and saturation mode operation for a NPN bipolar junction transistor. Consider the following transistor that has β=100. If it is desired that 1 mA current must be flowing in the collector at Vcc=1 V, what is the maximum value for RL that can be used before the transistor goes from active to saturation mode? [5pt]



active: $V_B > V_E$

$V_C > V_B$

sat: $V_B > V_E$

$V_B > V_C$

Edge of sat$^n$:

$V_B = V_C$

$V_B = 1V - I_B \times 10k\Omega$

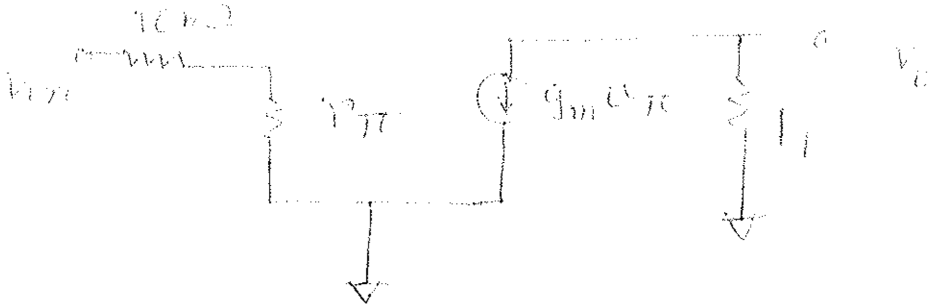
$V_C = 1.2V - I_C \times R_L$

$\Rightarrow 1.2 - 10^{-3} R_L = 1 - \frac{10^{-3}}{100} \times 10 \times 10^{3}$

$\Rightarrow 10^{-3} R_L = 1.2 - 0.9$

$\Rightarrow \boxed{R_L = 300\ \Omega}$ hence $\boxed{R_L \leq 300\Omega}$

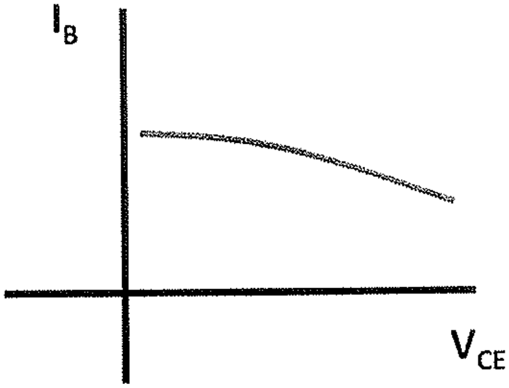
(c) Draw the small signal model for the BJT descried in part (b) assuming no Early effect. Indicate the numerical values and units for all the components of this small signal model. [5 pt]



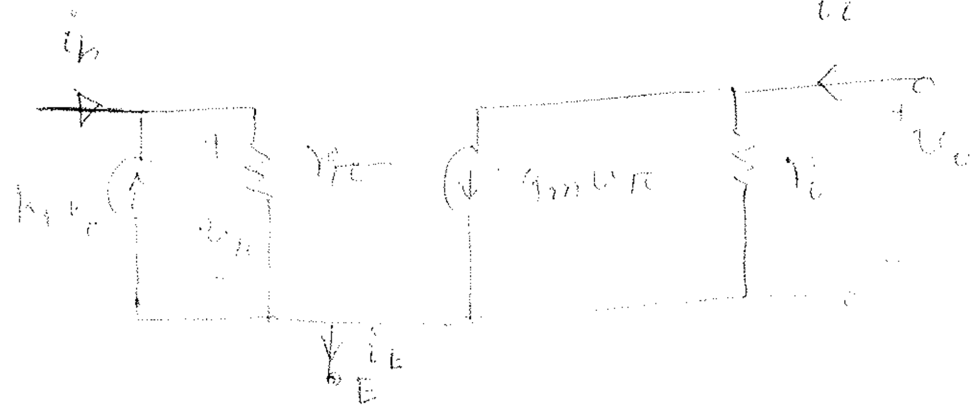
$$g_m = \frac{I_c}{V_T} = \frac{10^{-3}}{26\times10^{-3}} = \frac{1}{26\ \Omega}$$

$$r_\pi = \beta/g_m = \frac{100}{1/26} = 2600\ \Omega$$

(d) Early effect/ base width modulation causes the collector current to increase with collector-emitter voltage, $V_{CE}$. Normally it is assumed that the base current remains independent of $V_{CE}$. However, if the depletion region spreads a long distance into the base, it can reduce the base current by recombination of injected holes with the negative immobile charges in the base region (for a NPN transistor). This is shown in the following figure. Considering this decrease of $I_B$ with $V_{CE}$, derive and draw complete (including conventional early effect and the new phenomenon) the small-signal model of a NPN transistor. [6 pt]



Since $\frac{\partial I_B}{\partial v_{CE}} \neq 0$, we need to add a controlled source in the input circuit



$k_1 = \frac{\partial I_B}{\partial v_{CE}}$