University of California, Berkeley
EECS 105

Fall 2003
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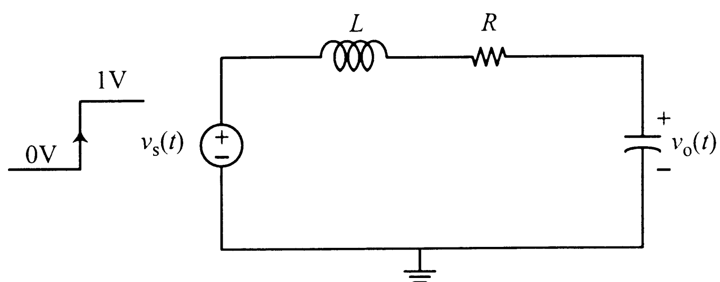
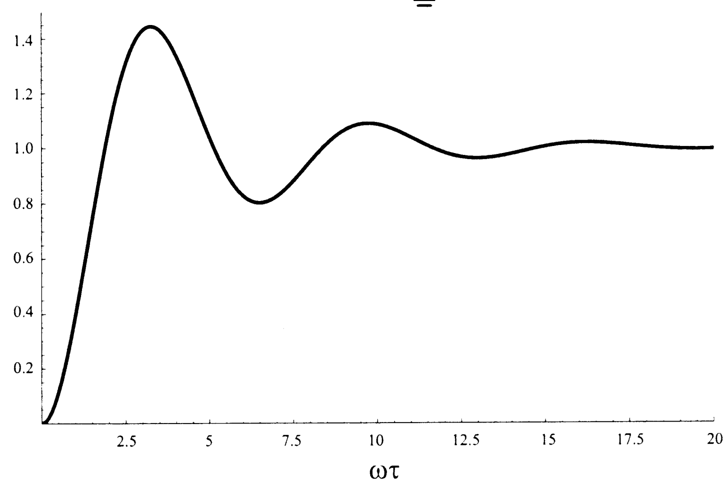
Midterm Exam (closed book)
Thursday, September 25, 2003

Helpful hints appear at the end of the exam.

1. (*20 points*) The step response of the series LCR circuit is shown below.





(a) Is the damping factor ζ greater than or less than unity?

$\zeta < 1$ (OVERSHOOT, RINGING)

(b) Qualitatively, describe the effect of decreasing the capacitance of the circuit.

$$Q = \frac{1}{\omega_0 RC} \qquad \omega_0 = \sqrt{\frac{1}{LC}}$$

$$Q = \frac{\sqrt{LC}}{RC} = \frac{\sqrt{L/C}}{R}$$

$C \downarrow \Rightarrow Q \uparrow$ MORE RINGING

(c) If it is desired to obtain a critically damped response, find the value of the series resistance R if $\sqrt{L/C} = 50\Omega$.

$$\zeta = 1 = \frac{1}{2Q} \qquad Q = \frac{1}{2} = \frac{\sqrt{L/C}}{R}$$

$$R = 100\Omega$$

(d) If the circuit is now driven with a sinusoidal voltage source of frequency $\omega \neq \omega_0$, find an expression for the power dissipation in the resistor R?

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

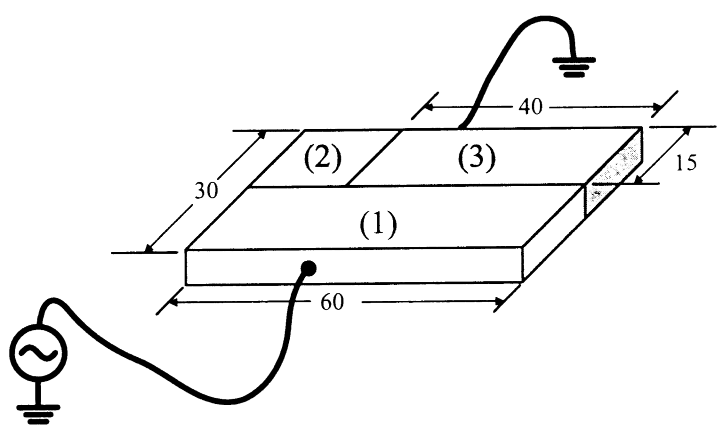
$$P = \frac{1}{2} Re\,(V I^*) = \frac{1}{2} Re\left(\frac{V V^*}{Z^*}\right)$$

$$= \frac{1}{2} |V|^2 \, Re\left(\frac{Z}{Z Z^*}\right) = \frac{1}{2} \frac{|V|^2 \, Re(Z)}{|Z|^2}$$

$$Re(Z) = R$$

$$|Z|^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

2. (*20 points*) For the resistor shown below



The sheet resistances of the region are $R_\square^1 = 10\text{k}\Omega/\text{sq}$, $R_\square^2 = 50\text{k}\Omega/\text{sq}$, $R_\square^3 = 30\text{k}\Omega/\text{sq}$.

(a) Calculate the resistance of the structure

$R_{eq} = R_1 + R_2 \| R_3 = 2.5k + 8.65k = 11.15k$

$R_1 = \frac{15}{60} 10k = 2.5k$

$R_2 = \frac{15}{20} 50k = 37.5k$

$R_3 = \frac{15}{40} 30k = 11.25k$

$R_2 \| R_3 = 8.65k$

$R_{eq} = 11.15k$

(b) At what voltage level does Ohm's law begin to fail, assuming that the carriers saturate when the electric field approaches 10^4V/cm?

$E = \frac{V_{\cancel{X}}}{\ell_1}$

$V_X = I R_X$

$I = V / R_{eq}$

V ACROSS REGION 2/3 IS HIGHEST

$V_X = \ell_X \cdot E = 10^4 \frac{V}{cm} 15\mu = 15\,V$

$I = \frac{15}{R_2 \| R_3}$

$V = I \cdot R_{eq} \cong 19.3\,V$

3. (*20 points*)A diffusion resistor is fabricated by doping Si with a group three element. The doping has a uniform concentration of 10^{18}cm^{-3}. The thickness of the diffusion region is unknown but the measured resistance of a long rectangular resistor with $L = 100\mu$ and $W = 5\mu$ is 3kΩ. (a) Estimate the free carrier electron and hole densities.

$N_A = 10^{18}\,cm^{-3} \Rightarrow p = 10^{18}\,cm^{-3}$ $\quad n = \frac{10^{20}}{10^{18}} = 10^2\,cm^{-3}$

(b) Calculate the thickness of the diffusion region (be careful with units of cm!).

$$R = \left(\frac{L}{W}\right)\frac{1}{\sigma t}$$

$$\sigma t = \left(\frac{L}{W}\right)\frac{1}{R} = \left(\frac{100}{5}\right)\frac{1}{3k} = \frac{20}{3}\ mS$$

$$\sigma \simeq p\mu q = 10^{18}\ cm^{-3} \times 160 \times \frac{cm^2}{V \cdot S} \times 1.6 \times 10^{-19}\ C$$

$$= 25.6\ S/cm$$

$$t = 2.6 \times 10^{-4}\ cm = 2.6\,\mu$$

(c) The same sample is now heated to extend the diffusion region. It is known that the diffusion region grows uniformly by 5μ, so the device now has a length of $L = 110\mu$, a width $W = 15\mu$, and a thickness that increases by 5μ. Calculate the resistance of the diffusion resistor.

$$V_1 = 100\mu \times 5\mu \times 2.6\mu = 1300\,\mu^3$$

$$V_2 = 110\mu \times 15\mu \times 7.6\mu = 12540\,\mu^3$$

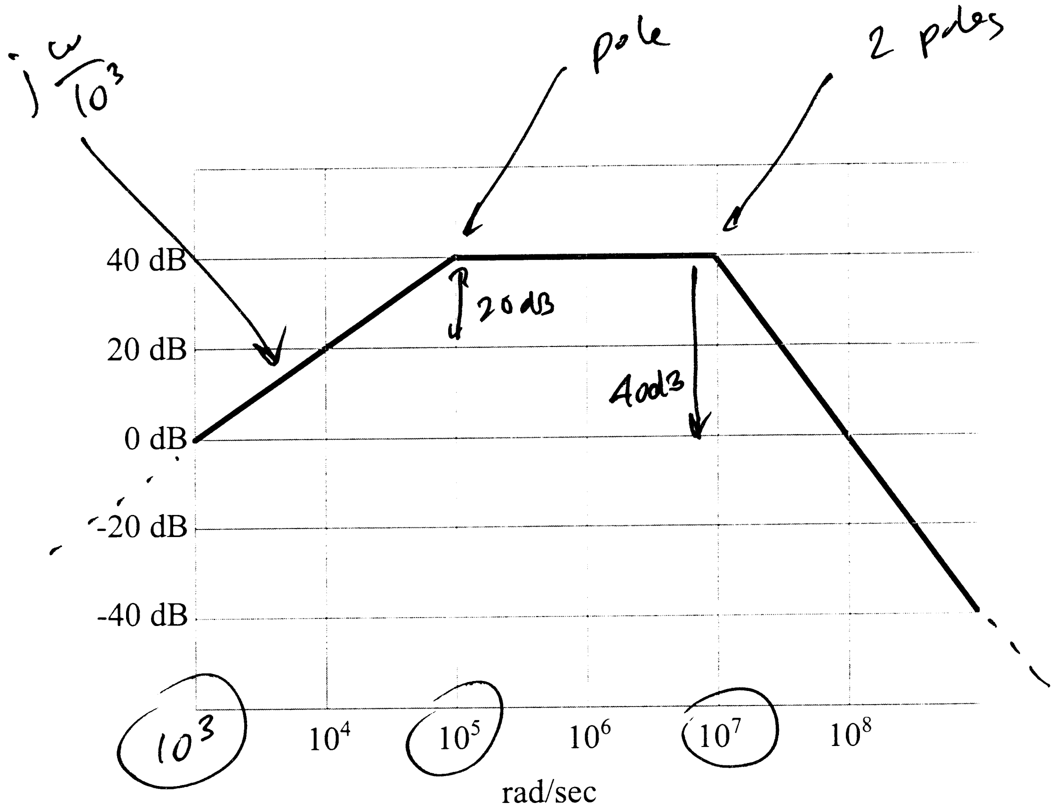
$$\frac{V_2}{V_1} \cong 10$$

$$N_A' = 10^{17}\ cm^{-3} \Rightarrow \mu_p = 330$$

$$\sigma = p\mu q = 5.28\ S/cm$$

$$R = \left(\frac{L}{W}\right)\frac{1}{\sigma t} = 1.8\ k\Omega$$

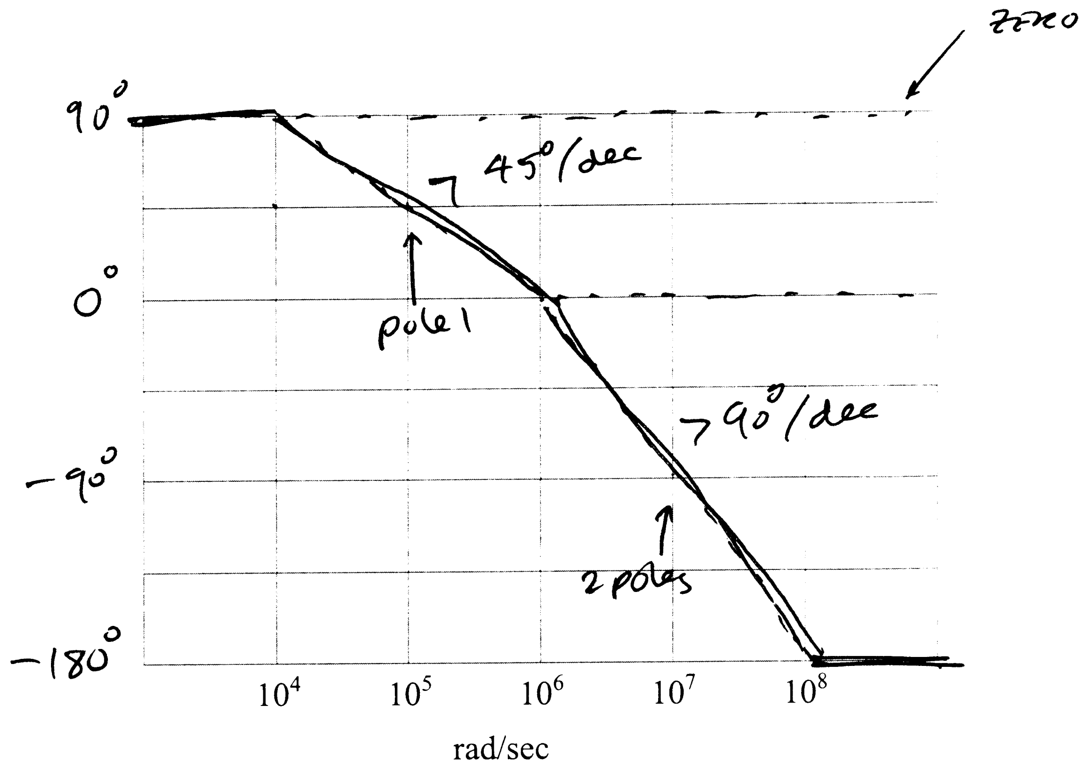
4. (*20 points*) Given the Bode plot shown below



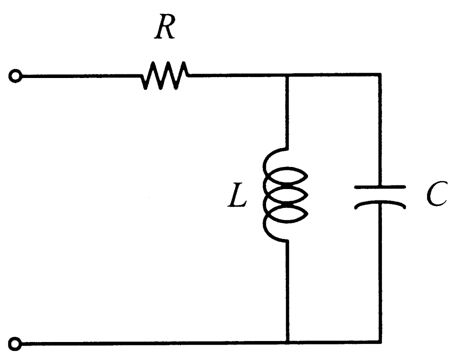
(a) write the transfer function.

$$H(j\omega) = \frac{\frac{j\omega}{10^3}}{\left(1+\frac{j\omega}{10^5}\right)\left(1+\frac{j\omega}{10^7}\right)^2}$$

(b) Draw the phase response assuming all poles and zero occur in the left half plane (in the provided graph).



5. (*20 points*) Given the circuit shown below



(a) Calculate the impedance transfer function. Identify the poles and zeros assuming that $\omega_0 L/R = 10$.

$$Z = R + j\omega L \,\|\, \frac{1}{j\omega C}$$

$$= R + \frac{j\omega L \cdot 1/j\omega C}{j\omega L + \frac{1}{j\omega C}} = R + \frac{j\omega L}{1 + (j\omega)^2 LC}$$

$$= \frac{R\left(1 + (j\omega)^2 LC\right) + j\omega L}{1 + (j\omega)^2 LC}$$

poles: $\pm j\omega_0$

$$\omega_0 \equiv \sqrt{\frac{1}{LC}} \qquad = R\,\frac{1 + j\omega\frac{L}{R} + (j\omega)^2 LC}{1 + (j\omega)^2 LC}$$

(b) The denominator for the transfer function is a second order function. What is the Q of the denominator of the transfer function?

$$Z(j\omega) = R\,\frac{1 + j\frac{\omega}{\omega_0}\left(\frac{\omega_0 L}{R}\right) + \left(j\frac{\omega}{\omega_0}\right)^2}{1 + \left(j\frac{\omega}{\omega_0}\right)^2}$$

$$2\zeta = \frac{\omega_0 L}{R} = 10 \qquad \zeta = 5 > 1$$

TWO DISTINCT ZEROS ON REAL AXIS

(b) $Q = \infty$

(c) What happens at frequency $\omega = \sqrt{LC}$? Explain qualitatively the operation of the circuit at this frequency.

L & C RESONATE AND PRODUCE AN OPEN

$$Z = R + Z_{L\|C} = R + \text{OPEN} = \infty$$

EQ CIRCUIT

$Z(\omega_0)$