CS 70
Fall 2004
Midterm 2
Rao

1. Short Answer: one, two, three… (20 pts)

1. The size of the sample space of throwing n labeled balls into m labeled bins.
2. The size of the sample space of the experiment of choosing k cards out of n different cards.
3. The number of permutations of n items.
4. The number of ways to pick n fruits from k varieties of fruits.
5. The number of ways to pick n fruits from k varieties of fruits where one picks at least $l < k$ different fruits.

2. Short Answer: Probability. (24 pts)

1. The expectation of a random variable that is 1 when a coin with heads probability p comes up heads.
2. The variance of the random variable above.
3. Best upper bound on the probability that a random variable with expectation μ and variance $\mu^2/4$ is 0.
4. The expectation of a random variable $Z = X_1 + X_2$ where X_1 and X_2 are independent random variables with expectation μ and variance σ^2.
5. The variance of Z.
6. Throw n balls into m bins, the expected number of bins with exactly three balls in them.

3. Short Answer: Some distributions. (12 pts)

1. The probability that a geometrically distributed variable with parameter p has value i.
2. The probability that a binomially distributed variable with parameters p and n has value i.
3. Recall that in the Central Limit Theorem, the distribution of the average of n samples converges to a normal distribution with some standard deviation (i.e., square root of the variance). If you wish to decrease the standard deviation of resulting normal distribution by a factor of two how many samples would you need?
4. Assume, the c percent confidence interval for a random variable with distribution $N(0,1)$ is [-x, +x], what is the c percent confidence interval for a random variable with distribution $N(\mu,\sigma^2)$?

4. Markov backwards? (14 pts)

1. Give a distribution for a random variable where the expectation is 1,000,000 and the probability that the random variable is zero is 99%.
2. Consider a random variable Y with expectation μ whose maximum value is $3\mu/2$, prove that the probability that Y is 0 is at most 1/3.

5. Sample Space. (15 pts)

You are given a bag containing three pancakes: One golden on both sides, one burnt on both sides, and one golden on one side and burnt on the other. You shake the bag, draw a pancake at random, look at one side and notice that it is golden. What is the probability that the other side is golden? Show your work.

6. Fake coins. (15 pts)

Suppose you are given a bag containing *n* unbiased coins. You are told that *n*-1 of these are normal coins, with heads on one side and tails on the other; however, the remaining coin has heads on both its sides.

1. Suppose you reach into the bag, pick out a coin uniformly at random, flip it and get a head. What is the (conditional) probability that this coin you chose is the fake (i.e., double-headed) coin?
2. Suppose you flip the coin *k* times after picking it (instead of just once) and see *k* heads. What is now the conditional probability that you picked the fake coin?
3. Suppose you wanted to decide whether the chosen coin was fake by flipping it *k* times; the decision procedure returns FAKE if all *k* flips come up heads, otherwise it returns NORMAL. What is the (unconditional) probability that this procedure makes an error?