CS 188 Midterm Exam, 2-330 pm, Mon Oct 19, 1998

1. (a) If you consider the flow field, then the FOE is the point where all the lines containing flow vectors intersect. This is the point that appears stationary.

   To get the time to collision, recall that if you consider the frame of reference in the image plane that has been shifted so that the FOE is at the origin, then the flow vector (u, v) at the image point (x, y) (with coordinates measured relative to FOE) is given by

   $$(u, v) = (\frac{Z'}{Z}x, \frac{Z'}{Z}y),$$

   where Z is the distance from the target to the camera, Z' is the normal-to-the-image-plane component of the camera's velocity with respect to the target. Notice that $\tau = Z/Z'$, and so $(u, v) = \frac{1}{\tau}(x, y)$. Hence the Time-to-collision can be found as the ratio of an image point's distance to the FOE to the point's apparent speed.

   (b) To go towards a target, make sure the target appears stationary (i.e., is at the FOE). The time to collision τ should tell you when to break.

   (c) Only if there is an object you know to be 100 feet away, so you can steer towards it. Optical flow (or monocular images in general) does not allow you to measure absolute distances.

2. (a) The length l_0 should satisfy the condition:

   $$0.2 \exp(-\frac{1}{2 \cdot 2^2}(l_0 - 10)^2) = 0.8 \exp(-\frac{1}{2 \cdot 2^2}(l_0 - 6)^2)$$

   (where 0.2 and 0.8 are the prior probabilities for each vehicle type).

   With a few algebraic manipulations, you get:

   $$\begin{aligned} 4 = 0.8/0.2 &= \frac{\exp(-\frac{1}{2 \cdot 2^2}(l_0 - 10)^2)}{\exp(-\frac{1}{2 \cdot 2^2}(l_0 - 6)^2)} \\ &= \exp(-\frac{1}{8}(l_0 - 10)^2 + \frac{1}{8}(l_0 - 6)^2) \\ &= \exp(-\frac{1}{8}(100 - 20l_0 - 36 + 12l_0)) \\ &= \exp(l_0 - 8) \end{aligned}$$

   from which you get

   $$l_0 = 8 + \ln 4 \approx 9.39\ feet.$$

(b) The perceptron will have two weights: one the multiplicative weight for the length, the other the threshold. The output of the perceptron will be

$$y = g(w_1 l + w_0) = \frac{1}{1 + \exp(-w_1 l - w_0)}$$

(c) Let's define

$$a = w_1 l + w_0.$$

Using the form of y given above, and the error

$$E = \frac{1}{2}(y - y^*)^2$$

(where y^* is the desired output), we get

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial a}\frac{\partial a}{\partial w_1} = (y - y^*)g'(a)l = (y - y^*)(y - y^2)l$$

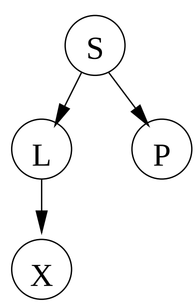
and, similarly,

$$\frac{\partial E}{\partial w_0} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial a}\frac{\partial a}{\partial w_0} = (y - y^*)(y - y^2).$$

Thus, for the 8ft truck, assuming the desired output of 1 for trucks and 0 for cars, we perform the updates

$$\begin{aligned} w_1 &\leftarrow w_1 - 8\alpha(y - 1)(y - y^2) \\ w_0 &\leftarrow w_0 - \alpha(y - 1)(y - y^2) \end{aligned}$$

3. Here is the belief net:



The probabilities you will need are: $P(S)$, $P(L|S)$, $P(P|S)$, and $P(X|L)$. You can find them by computing the proportion of people who smoke, occurence rates of lung cancer and poor stamina among people who smoke and among those who don't, and the proportion of spots found in X-ray images of people with cancer and those without.

4. Let's begin by computing the joint distribution. We'll have:

$$\begin{aligned}
Prob(A \wedge B) &= Prob(A)Prob(B|A) = 0.1 \cdot 0.9 = 0.09 \\
Prob(A \wedge \sim B) &= Prob(A)Prob(\sim B|A) = 0.1 \cdot 0.1 = 0.01 \\
Prob(\sim A \wedge B) &= Prob(\sim A)Prob(B| \sim A) = 0.9 \cdot 0.4 = 0.36 \\
Prob(\sim A \wedge \sim B) &= Prob(\sim A)Prob(\sim B| \sim A) = 0.9 \cdot 0.6 = 0.54
\end{aligned}$$

From these, we can find the prior for B and conditionals for B given A:

$$\begin{aligned}
Prob(B) &= 0.09 + 0.36 = 0.45 \\
Prob(A|B) &= Prob(A \wedge B)/Pr(B) = 0.09/0.45 = 0.2 \\
Prob(A| \sim B) &= Prob(A \wedge \sim B)/Pr(\sim B) = 0.01/0.55 = 1/55.
\end{aligned}$$