

Solutions for CS174 Midterm 2

1. (a) Because a degree d polynomial can have at most d roots. So the upper bound is d/M .
(b) Because a degree d polynomial can have at most d roots, if we can find $d+1$ distinct values of r such that the two polynomial agrees, that means $p(x) - q(x) = 0$ have $d + 1$ roots. Then we know for sure that they are identical.
2. (a) $m \approx k / \ln k$.
(b) $N \leq mk$. $k \approx m \ln k \approx m \ln(m \ln k) \approx m \ln m + m \ln(\ln k)$. So $k \leq 2m \ln m$, because $\ln k < m$. So $n < 2m^2 \ln m$.
(c) $N > mk/2$. Because $k \approx \frac{m}{2} \ln k \approx \frac{m}{2} \ln(\frac{m}{2} \ln k) > \frac{m}{2} (\ln m + \ln((\ln k)/2)) > \frac{m}{2} \ln m$. So $N > \frac{m^2}{4} \ln m$.
(d) $N = \Theta(m^2 \ln m)$.
3. (a) $t^2/n^2 = 1/2$.
(b) $P(t) = 1 - (1 - \frac{1}{4}P(t/2))^2$.
4. (a) There can be many answers for this. One is example is if we repeat the sequence of request $1, 2, \dots, k, k + 1$. Another example is that we have the sequence of request $1, 2, \dots, k$ first to warm up the cache, then we request the sequence $k + 1, 1, 2, \dots, k - 1$. So MIN will have 1 miss while LRU has k misses.
(b) Because in this case FIFO behaves the same way as LRU, so the competitive ratio will also be k .