# CS172, Spring 2001
Midterm 2

This midterm is open-book. There are 3 problems and 7 true/false questions. You have 80 minutes; the number of points assigned to each problem reflects the number of minutes expected to be spent on it, so there's a total of 80 points. We are not looking for rigidly formal construction proofs unless otherwise specified, but you should give sufficient detail to demonstrate that you can make the necessary construction based on constructions presented in class and/or in the textbook. Good luck!

1. (20 pts) Show that, if $P = NP$, there would exist a polynomial time algorithm that, given a graph G and a number k on input, outputs a simple path of length k in G if such a path exists.

2. We say that an undirected graph $G = (V, E)$ with $kn + 1$ nodes is a "k-daisy" if there is a collection of k "petals", $P_1, \ldots, P_n$, such that:

    (1) $\forall i, P_i \subseteq V$ and $|P_i| = n + 1$ (each petal is a set of $n + 1$ nodes)

    (2) $\exists c \in V$ s.t. $\forall i \neq j, P_i \cap P_j = \{c\}$ (there's a specific vertex c shared by all petals)

    (3) $\forall i$, there is a simple cycle in G through all vertices of P_i

Note that the above conditions implicitly force all vertices in the graph other than c to be included in exactly 1 petal (since, in addition to c, each of the k petals has n other nodes that it doesn't share with any other petal, and there's a total of $kn + 1$ nodes). For instance, the following graph is a 3-daisy (with the cycles through the 3 petals highlighted):
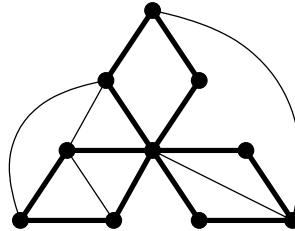
    a. (10 pts) Prove that the language $\text{DAISY} = \{\langle G, k \rangle | \text{the graph } G \text{ is a } k\text{-daisy}\}$ is NP-complete. You may not use the result from part (b).

    b. (12 pts) Prove that the language $5\text{-DAISY} = \{\langle G \rangle | \text{the graph } G \text{ is a } 5\text{-daisy}\}$ is NP-complete.

3. Define $L = \{\langle M \rangle | M$ is a TM that accepts at least 2 distinct strings$\}$.

    a. (8 pts) Show that A_{TM} is mapping-reducible to L.

    b. (10 pts) Use the Recursion Theorem to show that L is undecidable. You may not use your result from part (a).

    c. (8 pts) Show that L is Turing-recognizable.

4. (12 pts) True/False:

T	F	If COMPOSITE $= \{\langle N \rangle | N$ is a composite integer$\}$ is not NP-complete, then $P \neq NP$.
T	F	If a function $f : A \rightarrow B$ is a mapping reduction from A to B *and* is one-to-one, then B is mapping-reducible to A as well.
T	F	If A and B are in NP, then so is $A \cup B$.
T	F	If A and B are NP-complete, then so is $A \cap B$.
T	F	$CFL \subsetneq P$ (where CFL is the class of context-free languages).
T	F	The set of problems *to* which A_{TM} is mapping-reducible is uncountable (note that all such problems are necessarily undecidable).