Quiz 1

CS 172: Computability and Complexity, Fall 95

11am-12:30pm, 25 October

Notes: There are three questions on this quiz. Please answer each question in the space below it, using the back of the sheet to continue your answer if necessary. Please write clear, concise answers. If you are having problems with part of a question, leave it and try the next one.

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1. (a) Which of the following languages are regular? If the language is regular, provide a finite automaton or a regular expression for it. If not, give a proof using the pumping lemma.

i. The set of all 0-1 strings that contain either the substring 000 or the substring 111 (or both).
ii. The language $\{0^i 1^j 2^k : i = j \text{ or } j = k\}$.
iii. The set of all 0-1 strings of the form $0^i 1^j$, where i is odd and j is even.

(b) Give a context free grammar for each of the language in part (a).

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2. Which of the following statements are true? If the statement is true, give a short proof. If it is false, give a simple counterexample. You may assume without proof any result that was proven in class or on a homework, provided you state it clearly.

a. If L is regular and L is a subset of L, then L is regular.
b. If L_1, L_2, L_3 are regular, then the language U

$_{i=1}^{\infty}$ L_i is also regular.

- If L_1 and L_2 are both context free, then $L_1 \cap L_2$ is context free.
- If $L_1$is regular and L_2 is context free, then $L_1 \cap L_2$ is context free.
- If L_1 and L_2 are both recursively enumerable (r.e.), then $L_1 \cup L_2$ is also r.e.

Note: Remember that a Turing machine for an r.e. language may reject by computing forever.

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3. Recall from class that limiting the number of registers of a RAM to some fixed constant does not affect its computational power, since any language accepted by a RAM is also accepted by a RAM with only three registers. This problem investigates what happens when we limit the size of the registers, i.e., we allow them to contain only integers in some fixed range [*-N,,N*], where *N* is a constant.

a. Consider first a RAM in which both the number and the size of the registers are limited. Can any language that is accepted by a standard RAM also be accepted by such a RAM? Justify your answer carefully: wooly reasoning will receive little credit.
b. Now suppose we allow the RAM to have infinitely many registers, as in the standard model, and we limit only the size of the registers. Can any language that is accepted by a standard RAM also be accepted by such a RAM? Justify your answer carefully: woolly reasoning will receive little credit.