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David Wolfe
Try to keep your answers succinct.

1. (10 points) What is the language accepted by the following Turing machine? (Recall that " $1 / \mathrm{BR}$ " means on reading a 1 , write a Blank and move Right.)

2. Show that the following are true:
(a) (7 points) If $L$ is r.e., and $L \propto \bar{L}$, then $L$ is recursive.
(b) ( 7 points) If $L$ is recursive, then $L \propto 0^{*} 1^{*}$.
3. In this problem, we show that the following language is r.e., but not recursive:
$L_{\text {steps }}=\left\{\langle M\rangle:\right.$ there exist distinct strings $w_{1}$ and $w_{2}$ such that $M$ accepts $w_{1}$ and $w_{2}$ in the same number of steps $\}$
(a) (8 points) Show that $L_{\text {steps }}$ is r.e. (Hint: It may help to write language $L$ as $\left\{\langle M\rangle: \exists w_{1}, w_{2}, t\right.$ s.t. ... $\}$.)
(b) (6 points) Keeping in mind part (a), explain why we would not try to use each of the following possible reductions to show that $L_{\text {steps }}$ is not recursive.
i. Why won't $0^{*} 1^{*} \propto L_{\text {steps }}$ help?
ii. Why won't $L_{\text {steps }} \propto L_{u}$ help?
iii. Why won't $\overline{L_{u}} \propto L_{\text {steps }}$ help?
(c) (8 points) Prove that $L_{\text {steps }}$ is not recursive by showing that $L_{u} \propto L_{\text {steps }}$. (If you cannot do this part, at least be clear about what "you want" from your reduction.)
(d) (4 points) Is the complement of $L_{\text {steps }}$ r.e.? Why or why not?

| 1. | $/ 10$ |
| ---: | ---: |
| 2. | $/ 14$ |
| 3. | $/ 26$ |
| Total | $/ 50$ |

