CS170 Spring 1993
Midterm
Professor M. Blum

CLOSED BOOK. CALCULATORS ALLOWED.
You have two hours to complete this exam.
****DO ANY TWO OF THE THREE PROBLEMS. (Try to do all three if you can.)****

Problem #1 (Find An Element Above the Median)

Give upper and lower bounds on the number of comparisons to solve the following problems:

INPUT: An array $A=[a_1...a_N]$ of real numbers. N = even integer. The array is NOT sorted.
OUTPUT: An element a_1 contained in A that is greater than the MEDIAN, where the median is the biggest element in the bottom half.
EXAMPLE: A = [1, 5, 3, 6]; MEDIAN = 3; RETURN 5 or 6.

Use the decision tree model of computation. (Each comparison counts 1 step.)
Make your bounds as tight as you can make them, but no tighter.

Problem #2 (Celebrity Problem)

DEFINITION: A celebrity is someone whom everyone knows, but who knows no one (else).
THE PROBLEM: You are to determine if a party of N persons, N >= 2, has a celebrity by asking questions of the form "Do you know that person over there?"
STEPS: Each question counts one step. All other computations are free.
ASSUME: Each person (including the celebrity, if any) answers every (such) question asked of him, and answers it honestly.
YOUR MISSION: Give upper and lower bounds on the number of steps (questions) to determine if a party has a celebrity. Make your bounds as tight as you can make them, but no tighter.
HINT: Each answer to a question fills one entry of the MxN matrix $[a_{ij}]$ defined by:
a_{ij} = 1 if i knows j, 0 if i does not know j, -1 if i=j

Problem #3 (A Sorting Problem)

In this problem, f:Z+->Z+ is a given function. (see I below).
An algorithm: Consider the following algorithm for computing "FOO $[a_1...a_N]$":
INPUT: An array $[a_1...a_N]$ of reals; N contained in Z+ (positive integers)
OUTPUT: $[a_1...a_N]$ a permutation of the input.
BEGIN: 1 If Nthen sort input & return; else
2 2.1 Do FOO $[a_1...a_{f(N)}]$ [Foo of top f(N) elements.]
2.2 Do FOO $[a_{N-f(N)+1}...a_N]$ [Foo of bottom f(N) elements.]
2.3 Do FOO $[a_1...a_{f(N)}]$ [Foo of top f(N) elements.]
END

NOTES

I.

For which of the following choices of f does FOO sort the input array? PROVE YOUR ANSWERS
A) f(N) = N-1 B) f(N) = ceiling(2/3 N) C) f(N) = ceiling((N+1)/2)

II.

Analyze the running times of A, B, C (from part I). Let S(N) = number of steps FOO takes on input [a_1...a_N].
