1. **(10 points)** A cargo plane can carry a maximum of 100 tons and a maximum of 60 cubic meters of cargo. There are three materials that need to be carried, and the cargo company may choose to carry any amount of each, up to the maximum amount available in each.

   Material 1 has density 2 tons/cubic meter, maximum available amount 40 cubic meters, revenue $1,000 per cubic meter.

   Material 2 has density 1 ton/cubic meter, maximum available amount 30 cubic meters, revenue $1,200 per cubic meter.

   Material 3 has density 3 tons/cubic meter, maximum available amount 20 cubic meters, revenue $12,000 per cubic meter.

   Write a linear program that optimizes revenue while satisfying all the constraints.

2. **(10 points)** In class we proved that 3-coloring is NP-complete. Use this fact to prove that the problem of 3-coloring remains NP-complete even in the special case where the given graph is required to have an even number of vertices and an even number of edges.

   Reduction from:

   Proof of correctness:

3. (30 points) A *subsequence* of a given sequence of numbers can be thought of as the new sequence obtained when you cross out some of the numbers. For example, 1,2,5,6,9, a

subsequence of the sequence 3,1,2,17,5,6,9, is obtained by crossing out 3 and 17.

Consider the following problem: Given a sequence of n number $x_1, \ldots, x_n$, find the length of the longest increasing subsequence. i.e. a subsequence $y_1, \ldots, y_k$ with k as large as possible and such that $y_1 < y_2 < \ldots < y_k$.

Give a dynamic programming algorithm for this problem.

Dynamic programming recurrence and initialization:

Justification of correctness:

Running time and justification:

4. (20 points) Call an edge of a flow network *critical* if decreasing the capacity of this edge results in a decrease in the maximum flow. Give an algorithm that finds a critical edge in a network. Your algorithm should run as fast as maximum flow.

Algorithm:

Justification of correctness:

Running time and justification:

5. (20 points) True or False? Circle the correct answer. No explanation required. Points *will not* be subtracted for wrong answers.

T F If the frequency of x is strictly greater than the frequency of y, then Huffman coding will assign a codeword to x that is at least as long as the codeword assigned to y.

T F NP stands for non-polynomial time.

T F 2-coloring is known to be NP-complete.

T F Linear Programming $\in P$.

T F If the capacity of an edge in the min-cut is increased by 1 the maximum flow necessarily increases by 1.

T F The capacity of every cut is greater than or equal to the value of the maximum flow.

T F There is a polynomial time algorithm for finding the maximum independent set in a tree.

T F $P \neq NP \Rightarrow$ - $SAT \notin P$.

T F A divide and conquer algorithm that breaks a problem of size n into 2 problems of size n / 2 at the cost of n^2 steps is slower than a divide and conquer algorithm that breaks a problem of size n into 3 problems of size n / 2 at the cost of n steps.

T F Raising the 90^{th} roots of unity to the 5^{th} power yields the 18^{th} roots of unity.